11/4/2022

235 $\dfrac{\sqrt{3\cdot\sqrt{9^x}}}{81^{x-1}} = 9^{2x+3}$ $\left[-\frac{1}{5}\right]$

$$\frac{\left(3\cdot(3^{2x})^{\frac{1}{2}}\right)^{\frac{1}{2}}}{3^{4(x-1)}} = 3^{2(2x+3)}$$

$$\frac{\left(3\cdot 3^{x}\right)^{\frac{1}{2}}}{3^{4(x-1)}} = 3^{2(2x+3)}$$

$$\frac{3^{\frac{x+1}{2}}}{3^{4(x-1)}} = 3^{4x+6}$$

$$3^{\frac{x+1}{2}-4(x-1)} = 3^{4x+6}$$

↓ passo agli esponenti

$$\frac{x+1}{2} - 4(x-1) = 4x+6$$

$$x+1-8(x-1) = 8x+12$$

$$x-8x-8x = 12-1-8$$

$$-15x = 3$$

$$\boxed{x = -\frac{1}{5}}$$

247 $5^{x+1} \cdot 25^{x} = \sqrt{5^{1-x}} \cdot \sqrt{125}$ $\left[\frac{2}{7}\right]$

$$5^{x+1} \cdot 5^{2x} = 5^{\frac{1-x}{2}} \cdot 5^{\frac{3}{2}}$$

$$x + 1 + 2x = \frac{1-x}{2} + \frac{3}{2}$$

$$2x + 2 + 4x = 1 - x + 3$$

$$7x = 2$$

$$\boxed{x = \frac{2}{7}}$$

262 $\begin{cases} 4^{y^2} - 2^{4x} = 0 \\ \dfrac{625^x \cdot 25^x}{\sqrt{125}} = \sqrt{5}\left(\dfrac{1}{5}\right)^y \end{cases}$ $\left[\left(\frac{1}{2}; -1\right); \left(\frac{2}{9}; \frac{2}{3}\right)\right]$

$$\begin{cases} 2^{2y^2} = 2^{4x} \\ \dfrac{5^{4x} \cdot 5^{2x}}{5^{\frac{3}{2}}} = 5^{\frac{1}{2}} \cdot 5^{-y} \end{cases} \qquad \begin{cases} 2y^2 = 4x \\ 5^{4x+2x-\frac{3}{2}} = 5^{\frac{1}{2}-y} \end{cases}$$

$$\begin{cases} y^2 = 2x \\ 6x - \frac{3}{2} = \frac{1}{2} - y \end{cases} \qquad \begin{cases} (-6x+2)^2 = 2x \\ y = -6x + 2 \end{cases} \qquad \begin{cases} 36x^2 + 4 - 24x - 2x = 0 \\ // \end{cases}$$

$36x^2 - 26x + 4 = 0$ $\quad 18x^2 - 13x + 2 = 0$ $\quad \Delta = 169 - 144 = 25$

$$x = \frac{13 \pm 5}{36} = \begin{cases} \frac{8}{36} = \frac{2}{9} \\ \frac{18}{36} = \frac{1}{2} \end{cases}$$

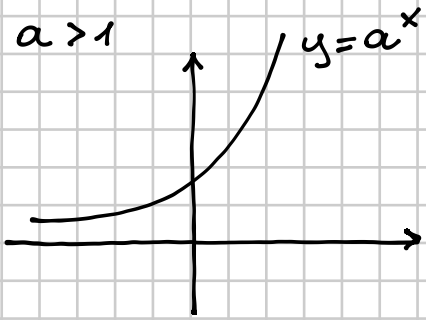
$$\begin{cases} x = \frac{2}{9} \\ y = -\cancel{6}^{2} \cdot \frac{2}{\cancel{9}_{3}} + 2 = -\frac{4}{3} + 2 = \frac{2}{3} \end{cases}$$

V

$$\begin{cases} x = \frac{1}{2} \\ y = -6 \cdot \frac{1}{2} + 2 = -1 \end{cases}$$

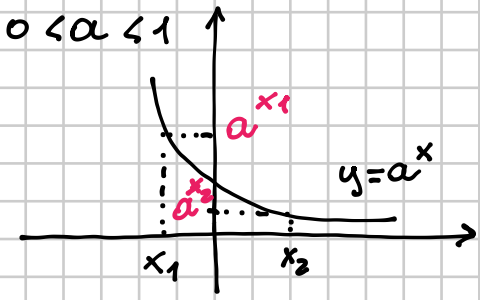
$\left(\frac{2}{9}, \frac{2}{3}\right)$ $\quad \left(\frac{1}{2}, -1\right)$

DISEQUAZIONI ESPONENZIALI



funz. CRESCENTE $\quad x_1 < x_2 \iff a^{x_1} < a^{x_2}$

ES. $\quad 2^{2x} < 2^3 \implies 2x < 3$



funz. DECRESCENTE $\quad x_1 < x_2 \iff a^{x_1} > a^{x_2}$

ES. $\left(\frac{1}{2}\right)^{2x} < \left(\frac{1}{2}\right)^3 \implies 2x > 3$

268 $\left(\frac{3}{2}\right)^x < \frac{27}{8}$ $\quad [x < 3]$

$\left(\frac{3}{2}\right)^x < \left(\frac{3}{2}\right)^3 \implies x < 3$ $\quad$ perché $\frac{3}{2} > 1$

271 $\left(\frac{1}{4}\right)^{x-1} < 64$ $\quad [x > -2]$

$\left(\frac{1}{4}\right)^{x-1} < \left(\frac{1}{4}\right)^{-3} \implies x - 1 > -3$ (INVERTO) perché $0 < \frac{1}{4} < 1$

$x > -2$

Rifatti:

$\left(4^{-1}\right)^{x-1} < 4^3 \qquad 4^{-x+1} < 4^3 \implies -x+1 < 3$ perché $4 > 1$

$-x < 2$

$x > -2$ come prima

278 $2^x \cdot 3^{x+1} \leq \frac{6^{3x}}{2}$ $\left[x \geq \frac{1}{2}\right]$

$$2 \cdot 2^x \cdot 3^{x+1} \leq 6^{3x}$$

$$2^{x+1} \cdot 3^{x+1} \leq 6^{3x}$$

$$(2 \cdot 3)^{x+1} \leq 6^{3x}$$

$$6^{x+1} \leq 6^{3x}$$

↓ perché $6 > 1$

$$x + 1 \leq 3x$$

$$-2x \leq -1$$

$$x \geq \frac{1}{2}$$

294 $\frac{-6}{2^x - 2} + \frac{9}{2^x - 1} < 0$ $[x < 0 \vee 1 < x < 2]$

$2^x = t$ $\qquad \frac{-6}{t-2} + \frac{9}{t-1} < 0$

$$\frac{-6(t-1) + 9(t-2)}{(t-2)(t-1)} < 0$$

$$\frac{-6t + 6 + 9t - 18}{(t-2)(t-1)} < 0$$

$$\frac{3t - 12}{(t-2)(t-1)} < 0$$

$$\frac{3t-12}{(t-2)(t-1)} < 0$$

$$\frac{\cancel{3}(t-4)}{(t-2)(t-1)} < 0$$

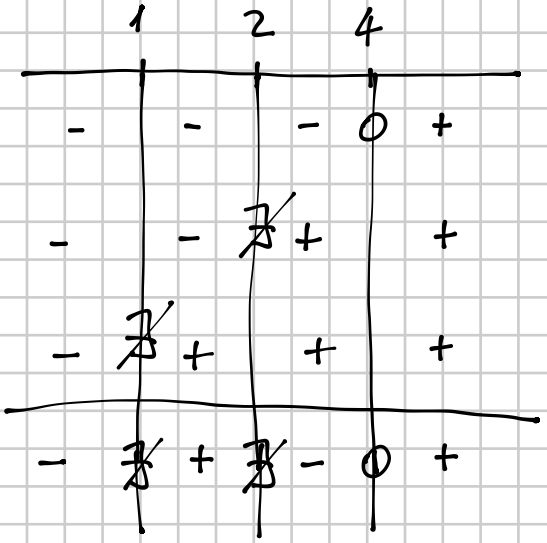
N_1

D_1 D_2

$t-4>0 \quad t>4$

$t-2>0 \quad t>2$

$t-1>0 \quad t>1$



$t<1 \quad \vee \quad 2<t<4$ $\qquad t=2^x$

$2^x<1 \quad \vee \quad 2<2^x<4$

$2^x<2^0 \quad \vee \quad 2^1<2^x<2^2$ ↘ $2>1$ mantengo la disuguaglianza passando agli esponenti

$$\boxed{x<0 \quad \vee \quad 1<x<2}$$