12/4/2022

279 $2 \cdot 3^{2x-1} + 9^{x+1} - 3^{2x+1} \leq \frac{60}{\sqrt[5]{3}}$ $\left[x \leq \frac{9}{10}\right]$

$$2 \cdot 3^{2x} \cdot 3^{-1} + 3^{2(x+1)} - 3^{2x} \cdot 3 \leq \frac{60}{\sqrt[5]{3}}$$

$$\frac{2}{3} \cdot 3^{2x} + 3^{2x} \cdot 3^2 - 3^{2x} \cdot 3 \leq \frac{60}{\sqrt[5]{3}}$$

$t = 3^{2x}$

$$\frac{2}{3} t + 9t - 3t \leq \frac{60}{\sqrt[5]{3}}$$

$$\frac{2t + 27t - 9t}{3} \leq \frac{60}{\sqrt[5]{3}}$$

$$\frac{20}{3} t \leq \frac{60}{\sqrt[5]{3}}$$

$$t \leq \frac{\overset{3}{\cancel{60}}}{\sqrt[5]{3}} \cdot \frac{3}{\cancel{20}}$$

$$t \leq 3^{2-\frac{1}{5}}$$

$$3^{2x} \leq 3^{2-\frac{1}{5}}$$

$$2x \leq 2 - \frac{1}{5}$$

$$2x \leq \frac{9}{5}$$

$$\boxed{x \leq \frac{9}{10}}$$

280 $\cancel{17}\cdot\sqrt{2^{x+1}} > \overset{2}{\cancel{34}}\cdot\sqrt[3]{4^{x-3}}$ $[x<9]$

$$2^{\frac{x+1}{2}} > 2\cdot 2^{\frac{2(x-3)}{3}}$$

$$2^{\frac{x+1}{2}} > 2^{\frac{2x-6}{3}+1}$$

$$\frac{x+1}{2} > \frac{2x-6}{3}+1$$

$$\frac{3x+3}{\cancel{6}} > \frac{4x-12+6}{\cancel{6}}$$

$$-x > -9$$

$$\boxed{x<9}$$

321 $\sqrt{2\cdot 6^x+7} \le 6^x+1$ $\left[x \ge \frac{1}{2}\right]$

$\sqrt{A(x)} \le B(x)$

$$\begin{cases} B(x)\ge 0 \\ A(x)\ge 0 \\ A(x)\le B^2(x) \end{cases}$$

$\Downarrow$ ho già che $A(x)\ge 0$ e $B(x)\ge 0$

$2\cdot 6^x+7 \le (6^x+1)^2$ $\qquad t=6^x$

$$2t+7 \le (t+1)^2$$

$$\cancel{2t}+7 \le t^2+1+\cancel{2t}$$

$$t^2 \ge 6$$

alternativa:

$6^{2x}\ge 6 \Rightarrow 2x\ge 1$

$\Downarrow$

$x \ge \frac{1}{2}$

$$t\le -\sqrt{6} \quad \vee \quad t\ge\sqrt{6}$$

$6^x \le -\sqrt{6} \quad \vee \quad 6^x \ge 6^{\frac{1}{2}} \Rightarrow \boxed{x\ge\frac{1}{2}}$

IMPOSSIBILE

324 $3^x - 9 < \sqrt{9^x - 9}$ $\quad [x \geq 1]$

$$\sqrt{9^x - 9} > 3^x - 9$$

$$\sqrt{A(x)} > B(x)$$

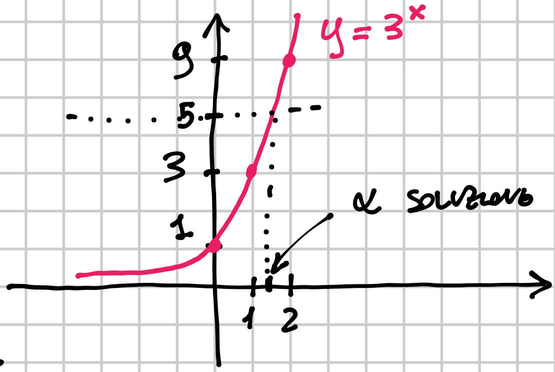
$$\begin{cases} B(x) < 0 \\ A(x) \geq 0 \end{cases} \vee \begin{cases} B(x) \geq 0 \\ A(x) > B^2(x) \end{cases}$$

(1) $\begin{cases} 3^x - 9 < 0 \\ 9^x - 9 \geq 0 \end{cases}$ $\vee$ (2) $\begin{cases} 3^x - 9 \geq 0 \\ 9^x - 9 > (3^x - 9)^2 \end{cases}$

(1) $\begin{cases} 3^x < 3^2 \\ 9^x \geq 9 \end{cases}$ $\begin{cases} x < 2 \\ x \geq 1 \end{cases}$ $\Rightarrow$ $1 \leq x < 2$

(2) $\begin{cases} 3^x \geq 3^2 \\ \cancel{3^{2x}} - 9 > \cancel{3^{2x}} - 18 \cdot 3^x + 81 \end{cases}$ $\begin{cases} x \geq 2 \\ 18 \cdot 3^x > 90 \end{cases}$ $\begin{cases} x \geq 2 \\ 2 \cdot 3^x > 10 \end{cases}$

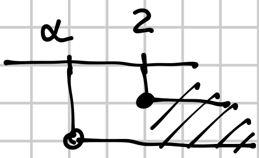
$\begin{cases} x \geq 2 \\ 3^x > 5 \end{cases}$



α soluzione di $3^x = 5$

$1 < \alpha < 2$

$\begin{cases} x \geq 2 \\ x > \alpha \end{cases}$



$\Downarrow$

$x \geq 2$

questo α si chiamerà LOGARITMO IN BASE 3 DI 5

$\log_3 5$

(1) $\vee$ (2)

$1 \leq x < 2 \quad \vee \quad x \geq 2$

$\Downarrow$

$$\boxed{x \geq 1}$$

346 $y = \sqrt{\dfrac{3^x - 1}{3^{-x} - 3}}$ $[-1 < x \leq 0]$

DETERMINARE IL DOMINIO

$$\frac{3^x - 1}{3^{-x} - 3} \geq 0 \qquad 3^x = t$$

perché $t > 0$

$$\frac{t-1}{\frac{1}{t} - 3} \geq 0 \qquad \frac{t-1}{\frac{1-3t}{t}} \geq 0 \qquad \frac{\cancel{t}(t-1)}{1-3t} \geq 0$$

N] $t - 1 > 0$ $\quad t > 1$

D] $1 - 3t > 0$ $\quad t < \frac{1}{3}$

		$\frac{1}{3}$		1	
N	$-$		$-$	0	$+$
D	$+$	$\nexists$	$-$		$-$
	$-$	$\nexists$	$+$	0	$-$

$$\frac{1}{3} < t \leq 1$$

$$\frac{1}{3} < 3^x \leq 1$$

$$3^{-1} < 3^x \leq 3^0$$

$$-1 < x \leq 0$$

$$D = (-1, 0]$$