**312** $\dfrac{3\cdot 3^{2x}-4\cdot 4^{2x}}{|-1+5^{x+1}|-4}<0$ $\quad\left[x<-\frac{1}{2}\vee x>0\right]$

(N: numeratore, D: denominatore)

$N>0 \qquad 3\cdot 3^{2x}-4\cdot 4^{2x}>0$

$3^{2x+1}>4^{2x+1}$

$\dfrac{3^{2x+1}}{4^{2x+1}}>1 \qquad \left(\dfrac{3}{4}\right)^{2x+1}>\left(\dfrac{3}{4}\right)^{0}$ perché $\frac{3}{4}<1$

$2x+1<0$

$x<-\frac{1}{2}$

$D>0 \qquad |-1+5^{x+1}|-4>0$

$|5^{x+1}-1|>4 \implies 5^{x+1}-1>4 \ \vee\ 5^{x+1}-1<-4$

$5^{x+1}>5 \ \vee\ 5^{x+1}<-3$

$x+1>1 \qquad \emptyset$ IMPOSSIBILE

$x>0$

SCHEMA FINALE

N $\quad x<-\frac{1}{2}$

D $\quad x>0$

| | $x<-\frac{1}{2}$ | $-\frac{1}{2}$ | | $0$ | $x>0$ |
|---|---|---|---|---|---|
| N | + | 0 | − | | − |
| D | − | | − | ∄ | + |
| | (−) | 0 | + | ∄ | (−) |

$$\boxed{x<-\frac{1}{2} \ \vee\ x>0}$$

**313** $\dfrac{9 \cdot 3^{-x}}{9^x + 3^{2x}} > \dfrac{27}{2}$ $\left[x < -\dfrac{1}{3}\right]$

$$\frac{3^2 \cdot 3^{-x}}{3^{2x} + 3^{2x}} > \frac{27}{2}$$

$$\frac{3^{2-x}}{2 \cdot 3^{2x}} > \frac{27}{2}$$

$$3^{2-x-2x} > 3^3$$

$$3^{2-3x} > 3^3$$

$$2 - 3x > 3$$

$$-3x > 1$$

$$\boxed{x < -\frac{1}{3}}$$

**323** $|16^x - 4| \geq 4 + 2 \cdot 4^x$ $[x \geq 1]$

$$|f(x)| \geq g(x) \iff f(x) \leq -g(x) \quad \vee \quad f(x) \geq g(x)$$

$$16^x - 4 \geq 4 + 2 \cdot 4^x \quad \vee \quad 16^x - 4 \leq -4 - 2 \cdot 4^x$$

$$4^{2x} - 2 \cdot 4^x - 8 \geq 0 \quad \vee \quad 4^{2x} + 2 \cdot 4^x \leq 0$$

$$4^{2x} - 2\cdot 4^x - 8 \geq 0 \quad \vee \quad 4^{2x} + 2\cdot 4^x \leq 0$$

$t = 4^x$ $\qquad\qquad$ $t = 4^x$

$$t^2 - 2t - 8 \geq 0 \quad \vee \quad t^2 + 2t \leq 0$$

$$(t-4)(t+2) \geq 0 \qquad t(t+2) \leq 0$$

$$t \leq -2 \ \vee \ t \geq 4 \qquad -2 \leq t \leq 0$$

$$4^x \leq -2 \ \vee \ 4^x \geq 4 \qquad -2 \leq 4^x \leq 0$$

IMPOSS. $\qquad$ IMPOSS. perché $4^x > 0 \ \forall x$

$$x \geq 1$$

$$\boxed{x \geq 1}$$

**332** $\dfrac{8^{1+x} + 8^x}{9} \geq 4^{1+2x} + \dfrac{16}{4^{1-2x}}$ $\qquad [x \leq -3]$

$$\frac{8\cdot 8^x + 8^x}{9} \geq 4\cdot 4^{2x} + \frac{\cancel{16}\,4}{\cancel{4}\cdot 4^{-2x}}$$

$$\frac{8^x \cancel{(8+1)}}{\cancel{9}} \geq 4\cdot 4^{2x} + 4\cdot 4^{2x}$$

$$2^{3x} \geq 8\cdot 2^{4x}$$

$$2^{3x} \geq 2^3\cdot 2^{4x} \qquad 2^{3x} \geq 2^{3+4x}$$

$$3x \geq 3 + 4x \qquad -x \geq 3$$

$$\boxed{x \leq -3}$$

**333** $\dfrac{2\cdot 4^x - 5\cdot 2^x + 2}{(25^x-5)\cdot(81\cdot 3^x - 3)} \le 0$ (N; $D_1$, $D_2$)

$\left[-3 < x \le -1 \lor \frac{1}{2} < x \le 1\right]$

$N > 0 \qquad 2\cdot 2^{2x} - 5\cdot 2^x + 2 > 0 \qquad t = 2^x$

$2t^2 - 5t + 2 > 0$

$\Delta = 25 - 16 = 9 \qquad t = \dfrac{5 \pm 3}{4} = \begin{cases} \frac{1}{2} \\ 2 \end{cases}$

$t < \frac{1}{2} \quad \lor \quad t > 2$

$2^x < 2^{-1} \quad \lor \quad 2^x > 2^1 \qquad x < -1 \quad \lor \quad x > 1$

$D_1 > 0 \quad 25^x - 5 > 0 \quad 5^{2x} > 5 \quad 2x > 1 \quad x > \frac{1}{2}$

$D_2 > 0 \quad 81\cdot 3^x - 3 > 0 \quad 3^{4+x} > 3 \quad 4 + x > 1 \quad x > -3$

$N > 0 \qquad x < -1 \quad \lor \quad x > 1$

$D_1 > 0 \qquad x > \frac{1}{2}$

$D_2 > 0 \qquad x > -3$

| | | $-3$ | | $-1$ | | $\frac{1}{2}$ | | $1$ | |
|---|---|---|---|---|---|---|---|---|---|
| N | + | | + | 0 | − | | − | 0 | + |
| $D_1$ | − | | − | | − | ∄ | + | | + |
| $D_2$ | − | ∄ | + | | + | | + | | + |
| | + | ∄ | ⊖ | 0 | + | ∄ | ⊖ | 0 | + |

$$\boxed{-3 < x \le -1 \quad \lor \quad \frac{1}{2} < x \le 1}$$

# IL NUMERO $e$

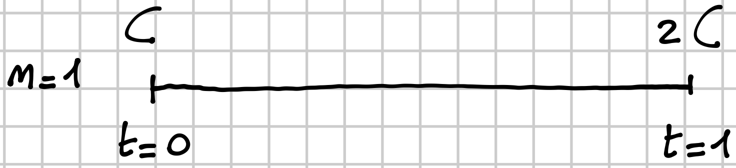
$e = 2,7182\ldots$ è la base privilegiata degli esponenziali

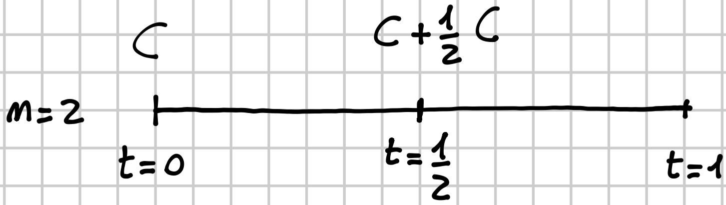
COSTANTE DI NEPERO

è un numero IRRAZIONALE

$C$ = capitale che investo al tempo $t=0$

Al tempo $t=1$ ho maturato un interesse $I = C$ (100%)




$t=0 \qquad C$

$t=\frac{1}{2} \qquad C\left(1+\frac{1}{2}\right)$

$t=1 \qquad C\left(1+\frac{1}{2}\right) + \frac{1}{2}C\left(1+\frac{1}{2}\right) =$

$= C\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2}\right) =$

$= C\left(1+\frac{1}{2}\right)^2$

$m=3$

$t=0 \qquad C$

$t=\frac{1}{3} \qquad C + \frac{1}{3}C = C\left(1+\frac{1}{3}\right)$

$t=\frac{2}{3} \qquad C\left(1+\frac{1}{3}\right) + \frac{1}{3}C\left(1+\frac{1}{3}\right) = C\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3}\right) = C\left(1+\frac{1}{3}\right)^2$

$t=1 \qquad C\left(1+\frac{1}{3}\right)^2 + \frac{1}{3}C\left(1+\frac{1}{3}\right)^2 = C\left(1+\frac{1}{3}\right)^2\left(1+\frac{1}{3}\right) = C\left(1+\frac{1}{3}\right)^3$

Dividendo in $n$ periodi, alla fine ottengo un MONTANTE (capitale + interessi) pari a

$$C\left(1+\frac{1}{n}\right)^{n}$$

Se $n$ è "grandissimo", significa che gli interessi maturati sono immediatamente investiti → CAPITALIZZAZIONE CONTINUA

n tende all'infinito

Quando $n$ cresce ($n \to \infty$) il fattore $\left(1+\frac{1}{n}\right)^{n}$ cresce stabilizzandosi sempre più vicino a un numero. Tale numero è $\boxed{e}$

$n = 1 \qquad \left(1+\frac{1}{n}\right)^{n} = 2$

$n = 100 \qquad \left(1+\frac{1}{100}\right)^{100} = 2{,}70481\ldots$

$n = 1000 \qquad \left(1+\frac{1}{1000}\right)^{1000} = 2{,}7169\ldots$

$n = 1000000 \qquad \left(1+\frac{1}{1000000}\right)^{1000000} = 2{,}7182\ldots$

↓

$e = 2{,}71828\ldots$

In regime di capitalizzazione continua, investendo un capitale $C$, con interesse 100%, ho alla fine un montante $C \cdot e$ (non $2C$).