

PROPRIETÀ DEI LOGARITMI

3/5/2022

$a > 0 \quad a \neq 1$ (BASE)

$x, y > 0$

- $\log_a(x \cdot y) = \log_a x + \log_a y$
- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
- $\log_a x^y = y \cdot \log_a x \quad (y \text{ qualiasi})$
- $\log_a 1 = 0$
- $\log_m x = \frac{\log_a x}{\log_a m} \quad m > 0 \quad m \neq 1$

DIMOSTRAZIONI

• $\boxed{\log_a x \cdot y = \log_a x + \log_a y}$

$$a^{\log_a x \cdot y} = a^{\log_a x + \log_a y}$$

$$a^{\log_a x \cdot y} = a^{\log_a x} \cdot a^{\log_a y}$$

$$x^y = x^y$$

• $\boxed{\log_a x^y = y \log_a x}$

$$a^{\log_a x^y} = a^{y \cdot \log_a x}$$

$$x^y = (a^{\log_a x})^y$$

$$x^y = x^y$$

CONSEGUENZA

$$\log_a \sqrt[n]{x} = \log_a x^{\frac{1}{n}} = \\ = \frac{1}{n} \log_a x$$

$$\log_m x = \frac{\log_a x}{\log_a m}$$

$$\log_a M \cdot \log_m x = \log_a x$$

$$(\log_m x) \cdot \log_a M = \log_a x$$

$$\log_a M^{\log_m x} = \log_a x$$

$$\log_a x = \log_a x$$

OSSERVAZIONE

$$\begin{aligned}\log_a \frac{x}{y} &= \log_a (x \cdot y^{-1}) = \log_a x + \log_a y^{-1} = \\ &= \log_a x + (-1) \cdot \log_a y = \\ &= \log_a x - \log_a y\end{aligned}$$

NOTA ZIONI

$$\text{Log} = \log_{10} \quad \ln = \log_e \quad e \approx 2,71 \quad \text{costante di NEPERO}$$

Log è AMBITUO \rightarrow a volte è \log_{10} \rightarrow a volte è \ln (per il libro è \log_{10})

A volte log si abbrevia in lg

82

$$\log_5(3ab^2)$$

$$[\log_5 3 + \log_5 a + 2\log_5 b]$$

Applicare le
proprietà dei
logaritmi

83

$$\log \frac{3\sqrt{a}}{b}$$

$$\left[\log 3 + \frac{1}{2} \log a - \log b \right]$$

84

$$\log_2 \left(\frac{2 \cdot \sqrt[3]{2}}{\sqrt{2}} \right)$$

$$\left[\frac{5}{6} \right]$$

82

$$\begin{aligned} \log_5(3ab^2) &= \log_5 3 + \log_5 a + \log_5 b^2 = \\ &= \log_5 3 + \log_5 a + 2 \log_5 b \end{aligned}$$

83

$$\begin{aligned} \log \frac{3\sqrt{a}}{b} &= \log(3\sqrt{a}) - \log b = \log 3 + \log \sqrt{a} - \log b = \\ &= \log 3 + \frac{1}{2} \log a - \log b \end{aligned}$$

84

$$\log_2 \frac{2 \cdot \sqrt[3]{2}}{\sqrt{2}} = \log_2 2 + \log_2 \sqrt[3]{2} - \log_2 \sqrt{2} =$$

$$= 1 + \log_2 2^{\frac{1}{3}} - \log_2 2^{\frac{1}{2}} =$$

$$= 1 + \frac{1}{3} \log_2 2 - \frac{1}{2} \log_2 2 =$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{6+2-3}{6} = \frac{5}{6}$$

111

$$\log_2(x+1) + 5\log_2(x-1) - 4\log_2(x^2-1) =$$

Scrivere sotto forma di un unico logaritmo

$$= \log_2(x+1) + \log_2(x-1)^5 - \log_2(x^2-1)^4 =$$

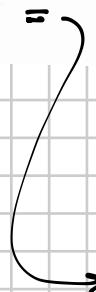
$$= \log_2 \frac{(x+1)(x-1)^5}{(x^2-1)^4} = \log_2 \frac{\cancel{(x+1)}(x-1)^5}{\cancel{(x-1)^4}(x+1)^4} = \log_2 \frac{x-1}{(x+1)^3}$$

150

$$\log_4 7 \cdot \log_7 16 =$$

[2]

$$\log_m x = \frac{\log_a x}{\log_a m}$$



$$= \cancel{\log_4 7} \cdot \frac{\log_4 16}{\cancel{\log_4 7}} = \log_4 4^2 = 2$$

Scrivere con un unico logaritmo:

134

$$\log_3 7 - 1$$

[$\log_3 \frac{7}{3}$]**135**

$$\log_2 3 + 2 - \log_2 12$$

[0]

$$\boxed{134} \quad \log_3 7 - 1 = \log_3 7 - \log_3 3 = \log_3 \frac{7}{3}$$

$$\boxed{135} \quad \log_2 3 + 2 - \log_2 12 = \log_2 3 + (2) \cdot \log_2 2 - \log_2 12 =$$

$$= \log_2 3 + \log_2 2^2 - \log_2 12 = \log_2 \frac{3 \cdot 4}{12} = \log_2 1 = 0$$