PROPRIETÀ DEI LOGARITMI

3/5/2022

$a > 0 \quad a \neq 1$ (BASE)

$x, y > 0$

- $\log_a (x \cdot y) = \log_a x + \log_a y$
- $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
- $\log_a x^y = y \cdot \log_a x \qquad$ (y qualsiasi)
- $\log_a 1 = 0$
- $\log_n x = \dfrac{\log_a x}{\log_a n} \qquad n > 0 \quad n \neq 1$

DIMOSTRAZIONI

- $\boxed{\log_a x \cdot y = \log_a x + \log_a y}$

$$a^{\log_a xy} = a^{\log_a x + \log_a y}$$

$$a^{\log_a xy} = a^{\log_a x} \cdot a^{\log_a y}$$

$$xy = xy$$

- $\boxed{\log_a x^y = y \log_a x}$

$$a^{\log_a x^y} = a^{y \cdot \log_a x}$$

$$x^y = \left(a^{\log_a x}\right)^y$$

$$x^y = x^y$$

CONSEGUENZA

$$\log_a \sqrt[n]{x} = \log_a x^{\frac{1}{n}} =$$

$$= \frac{1}{n} \log_a x$$

- $\boxed{\log_n x = \dfrac{\log_a x}{\log_a n}}$

$$\log_a n \cdot \log_n x = \log_a x$$

$$\log_n x \cdot \log_a n = \log_a x$$

$$\log_a n^{\log_n x} = \log_a x$$

$$\log_a x = \log_a x$$

OSSERVAZIONE

$$\log_a \frac{x}{y} = \log_a (x \cdot y^{-1}) = \log_a x + \log_a y^{-1} =$$

$$= \log_a x + (-1) \cdot \log_a y =$$

$$= \log_a x - \log_a y$$

NOTAZIONI

$\mathrm{Log} = \log_{10}$ $\quad$ $\ln = \log_e$ $\quad$ $e \simeq 2,71$ COSTANTE DI NEPERO

log è AMBIGUO ↗ a volte è $\log_{10}$ ↘ a volte è ln $\quad$ (per il libro è $\log_{10}$)

A volte log si abbrevia in lg

82 $\log_5(3ab^2)$ $[\log_5 3 + \log_5 a + 2\log_5 b]$

83 $\log\frac{3\sqrt{a}}{b}$ $\left[\log 3 + \frac{1}{2}\log a - \log b\right]$

84 $\log_2\left(\frac{2\cdot\sqrt[3]{2}}{\sqrt{2}}\right)$ $\left[\frac{5}{6}\right]$

Applicare le proprietà dei logaritmi

82

$$\log_5(3ab^2) = \log_5 3 + \log_5 a + \log_5 b^2 =$$

$$= \log_5 3 + \log_5 a + 2\log_5 b$$

83

$$\log\frac{3\sqrt{a}}{b} = \log(3\sqrt{a}) - \log b = \log 3 + \log\underbrace{\sqrt{a}}_{a^{\frac{1}{2}}} - \log b =$$

$$= \log 3 + \frac{1}{2}\log a - \log b$$

84

$$\log_2\frac{2\cdot\sqrt[3]{2}}{\sqrt{2}} = \log_2 2 + \log_2\sqrt[3]{2} - \log_2\sqrt{2} =$$

$$= 1 + \log_2 2^{\frac{1}{3}} - \log_2 2^{\frac{1}{2}} =$$

$$= 1 + \frac{1}{3}\log_2 2 - \frac{1}{2}\log_2 2 =$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{6+2-3}{6} = \frac{5}{6}$$

111 $\log_2(x+1) + 5\log_2(x-1) - 4\log_2(x^2-1) =$

Scrivere sotto forma di un unico logaritmo

$$= \log_2(x+1) + \log_2(x-1)^5 - \log_2(x^2-1)^4 =$$

$$= \log_2 \frac{(x+1)(x-1)^5}{(x^2-1)^4} = \log_2 \frac{\cancel{(x+1)}\cancel{(x-1)^5}}{\cancel{(x-1)^4}(x+1)^{\cancel{4}\,3}} = \log_2 \frac{x-1}{(x+1)^3}$$

150 $\log_4 7 \cdot \log_7 16 =$ [2]

$$\log_m x = \frac{\log_a x}{\log_a m}$$

$$= \cancel{\log_4 7} \cdot \frac{\log_4 16}{\cancel{\log_4 7}} = \log_4 4^2 = 2$$

Scrivere con un unico logaritmo:

134 $\log_3 7 - 1$ $\left[\log_3 \frac{7}{3}\right]$

135 $\log_2 3 + 2 - \log_2 12$ [0]

$\boxed{134}$ $\log_3 7 - 1 = \log_3 7 - \log_3 3 = \log_3 \frac{7}{3}$

$\boxed{135}$ $\log_2 3 + 2 - \log_2 12 = \log_2 3 + 2 \cdot \log_2 2 - \log_2 12 =$

$$= \log_2 3 + \log_2 2^2 - \log_2 12 = \log_2 \frac{3 \cdot 4}{12} = \log_2 1 = 0$$