485 $\ln x + \ln^2 x < 0$ $\left[\frac{1}{e} < x < 1\right]$

C.E. $x > 0$

$t = \ln x$ $\qquad t + t^2 < 0$

$t(t+1) < 0$ $\qquad t = -1$, $t = 0$

$-1 < t < 0$

$-1 < \ln x < 0$

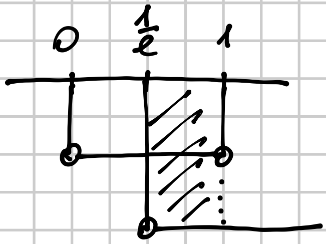
$\ln x < \ln 1$

$$\begin{cases} \ln x < 0 \\ \ln x > -1 \end{cases} \qquad \begin{cases} 0 < x < 1 \\ x > e^{-1} \end{cases}$$

$\ln x > -\ln e$

$\ln x > \ln e^{-1}$

$$\begin{cases} 0 < x < 1 \\ x > \frac{1}{e} \end{cases}$$



$$\boxed{\frac{1}{e} < x < 1}$$

488 $3-\ln|x|<0$ $[x<-e^3 \vee x>e^3]$

C.E. $x \neq 0$

$-\ln|x| < -3$

$\ln|x| > 3$

$\ln|x| > 3 \cdot \underbrace{\ln e}_{1}$

$\ln|x| > \ln e^3$

$|x| > e^3$ $\boxed{x < -e^3 \vee x > e^3}$

493 $\left(\log_{\frac{1}{4}} x\right)^2 + \frac{5}{2}\log_{\frac{1}{4}} x > \frac{3}{2}$ $\left[0<x<\frac{1}{2} \vee x>64\right]$

C.E. $x>0$

$\log_{\frac{1}{4}} x = t$

$t^2 + \frac{5}{2}t - \frac{3}{2} > 0$

$2t^2 + 5t - 3 > 0$ $\Delta = 25 + 24 = 49$

$t = \frac{-5 \pm 7}{4} = \begin{cases} -3 \\ \frac{1}{2} \end{cases}$

$t < -3 \vee t > \frac{1}{2} \Rightarrow \log_{\frac{1}{4}} x < -3 \vee \log_{\frac{1}{4}} x > \frac{1}{2}$

$\log_{\frac{1}{4}} x < -3 \cdot \log_{\frac{1}{4}} \frac{1}{4} \vee \log_{\frac{1}{4}} x > \frac{1}{2} \log_{\frac{1}{4}} \frac{1}{4}$

$\log_{\frac{1}{4}} x < \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{-3} \vee \log_{\frac{1}{4}} x > \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{\frac{1}{2}}$

$x > \left(\frac{1}{4}\right)^{-3} \vee \underbrace{0 <}_{C.E.} x < \left(\frac{1}{4}\right)^{\frac{1}{2}}$

$0 < x < \sqrt{\frac{1}{4}} \vee x > 4^3$

$\boxed{0 < x < \frac{1}{2} \vee x > 64}$

556 $3 \cdot 2^x + 2^{x+1} = 19$ $\left[\dfrac{\log 19 - \log 5}{\log 2}\right]$

$$3 \cdot 2^x + 2^x \cdot 2 = 19$$

$$2^x(3+2) = 19$$

$$5 \cdot 2^x = 19$$

$$2^x = \frac{19}{5}$$

applico a tutte e due le parti $\log_2$

$$\log_2 2^x = \log_2\left(\frac{19}{5}\right)$$

$$x = \log_2 \frac{19}{5}$$

ALTERNATIVO

$$2^x = 2^{\log_2 \frac{19}{5}}$$

$$\Downarrow$$

$$x = \log_2 \frac{19}{5}$$

Il libro continua e scrive $\log_2 \frac{19}{5}$ come combinazione di log in base 10

$$\log_2 \frac{19}{5} = \frac{\log \frac{19}{5}}{\log 2} = \frac{\log 19 - \log 5}{\log 2}$$

571 $7 \cdot 2^x + \dfrac{5}{2^x} = \dfrac{117}{4}$ $\left[2; \dfrac{\log 5 - \log 28}{\log 2}\right]$

$t = 2^x \qquad 7t + \dfrac{5}{t} = \dfrac{117}{4} \qquad \dfrac{28t^2 + 20}{\cancel{4t}} = \dfrac{117t}{\cancel{4t}}$

$28t^2 - 117t + 20 = 0$

$\Delta = 117^2 - 20 \cdot 28 \cdot 4 = 11449 = 107^2$

$t = \dfrac{117 \pm 107}{56} = \begin{cases} \dfrac{10}{56} = \dfrac{5}{28} \\ \dfrac{224}{56} = 4 \end{cases}$

$t = \dfrac{5}{28} \quad \vee \quad t = 4$

$2^x = \dfrac{5}{28} \quad \vee \quad 2^x = 4$

$x = \log_2 \dfrac{5}{28} \quad \vee \quad x = 2$

$x = \dfrac{\log \frac{5}{28}}{\log 2}$

$$\boxed{x = \dfrac{\log 5 - \log 28}{\log 2} \quad \vee \quad x = 2}$$

615 $3^{2x} - 4 \geq 0$ $\left[x \geq \frac{\log 4}{2\log 3}\right]$

$3^{2x} \geq 4$

$2x \geq \log_3 4$

$x \geq \frac{1}{2}\log_3 4$ $\qquad x \geq \frac{1}{2}\frac{\log 4}{\log 3} = \frac{\log 2^2}{2\log 3} = \frac{\cancel{2}\log 2}{\cancel{2}\log 3}$

$x \geq \frac{\log 2}{\log 3}$