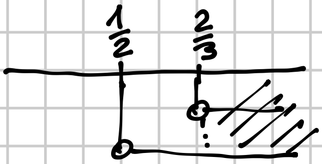
18/5/2022

527 $\log_2(3x-2) + \log_{\frac{1}{2}}(2x-1) \leq 2$ $\quad \left[x > \frac{2}{3}\right]$

C.E. $\begin{cases} 3x-2>0 \\ 2x-1>0 \end{cases}$ $\quad \begin{cases} x > \frac{2}{3} \\ x > \frac{1}{2} \end{cases}$



$x > \frac{2}{3}$

$$\log_2(3x-2) + \frac{\log_2(2x-1)}{\underbrace{\log_2 \frac{1}{2}}_{-1}} \leq 2 \cdot \overbrace{\log_2 2}^{1}$$

$$\log_2(3x-2) - \log_2(2x-1) \leq \log_2 2^2$$

$$\log_2 \frac{3x-2}{2x-1} \leq \log_2 4$$

siccome $2>1$ mantengo la disug.

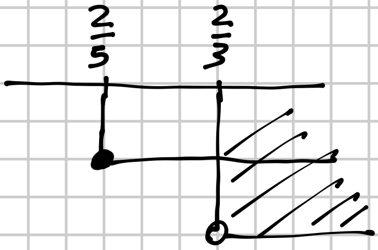
$$\frac{3x-2}{2x-1} \leq 4$$

$3x-2 \leq 4(2x-1)$ perché nelle C.E. c'è $2x-1>0$

$$3x-2 \leq 8x-4$$

$$-5x \leq -2$$

$$\begin{cases} x \geq \frac{2}{5} \\ x > \frac{2}{3} \end{cases}$$



$$\boxed{x > \frac{2}{3}}$$

521 $(\log_2 x)^2 - 8 \geq 4\log_2 \sqrt{x}$ $\left[0 < x \leq \frac{1}{4} \vee x \geq 16\right]$

NOTAZIONI

$\log^2 x = (\log x)^2$

$\log x^2 = \log(x^2)$

C.E. $x > 0$

$$(\log_2 x)^2 - 8 \geq 4\log_2 x^{\frac{1}{2}}$$

$$\log_2^2 x - 8 \geq \cancel{4}^{2} \cdot \frac{1}{\cancel{2}} \log_2 x$$

$\log_2 x = t$

$$t^2 - 8 \geq 2t$$

$$t^2 - 2t - 8 \geq 0$$

$$(t-4)(t+2) \geq 0$$

$$t \leq -2 \quad \vee \quad t \geq 4$$

$$\log_2 x \leq -2 \quad \vee \quad \log_2 x \geq 4$$

$$\log_2 x \leq -2 \cdot \log_2 2 \quad \vee \quad \log_2 x \geq 4 \cdot \log_2 2$$

$$\log_2 x \leq \log_2 2^{-2} \quad \vee \quad \log_2 x \geq \log_2 2^4$$

$$0 < x \leq 2^{-2} \quad \vee \quad x \geq 2^4$$

$$\boxed{0 < x \leq \frac{1}{4} \quad \vee \quad x \geq 16}$$

596 $3^{x+1}-2\cdot 3^{x}+3^{x+2}=5^{x-1}$ $\left[\dfrac{2\log 5+\log 2}{\log 5-\log 3}\right]$

$$3^x\cdot 3-2\cdot 3^x+3^x\cdot 3^2=5^x\cdot\frac{1}{5}$$

$$3^x(3-2+9)=5^x\cdot\frac{1}{5}$$

$$3^x\cdot 10=\frac{5^x}{5}$$

$$\frac{3^x}{5^x}=\frac{1}{50}$$

$$\left(\frac{3}{5}\right)^x=\frac{1}{50} \qquad x=\log_{\frac{3}{5}}\frac{1}{50}=\frac{\log\frac{1}{50}}{\log\frac{3}{5}}=$$

$$=\frac{\overbrace{\log 1}^{0}-\log 50}{\log 3-\log 5}=$$

$$=\frac{-\log(5^2\cdot 2)}{\log 3-\log 5}=\frac{\log 5^2+\log 2}{\log 5-\log 3}=$$

$$=\boxed{\frac{2\log 5+\log 2}{\log 5-\log 3}}$$

601 $3^{\frac{x+1}{2}} \cdot 7^{x-1} = \dfrac{1}{49^x \cdot 9^x}$ $\left[\dfrac{2\ln 7 - \ln 3}{5\ln 3 + 6\ln 7}\right]$

$$3^{\frac{x+1}{2}} \cdot 7^{x-1} = \frac{1}{7^{2x} \cdot 3^{2x}}$$

$$3^{\frac{x+1}{2}} \cdot 3^{2x} = \frac{1}{7^{x-1} \cdot 7^{2x}}$$

$$3^{\frac{x+1}{2}+2x} = 7^{1-x-2x}$$

$$3^{\frac{x+1+4x}{2}} = 7^{1-3x} \qquad 3^{\frac{5x+1}{2}} = 7^{1-3x}$$

$$3^{f(x)} = 7^{g(x)}$$

$$\ln 3^{f(x)} = \ln 7^{g(x)}$$

$$f(x) \cdot \ln 3 = g(x) \cdot \ln 7$$

$$\ln 3^{\frac{5x+1}{2}} = \ln 7^{1-3x}$$

$$\frac{5x+1}{2} \cdot \ln 3 = (1-3x) \cdot \ln 7$$

$$(5x+1) \cdot \ln 3 = 2(1-3x) \cdot \ln 7$$

$$5\ln 3 \cdot x + \ln 3 = 2\ln 7 - 6\ln 7 \cdot x$$

$$5\ln 3 \cdot x + 6\ln 7 \cdot x = 2\ln 7 - \ln 3$$

$$x(5\ln 3 + 6\ln 7) = 2\ln 7 - \ln 3$$

$$\boxed{x = \frac{2\ln 7 - \ln 3}{5\ln 3 + 6\ln 7}}$$