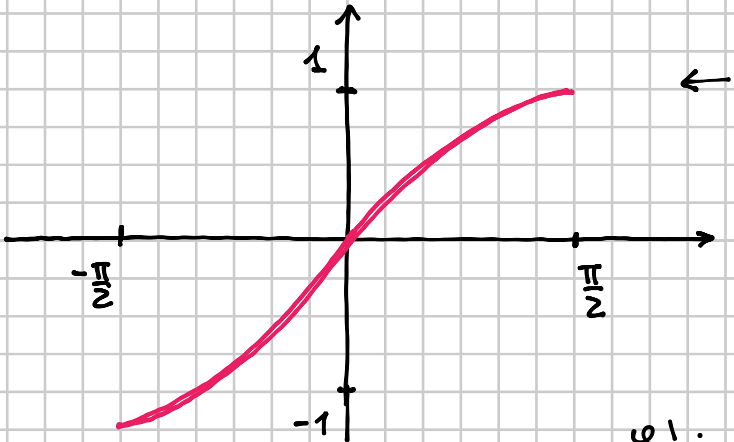
5/10/2022

$y = \sin x$ NON è invertibile perché non è iniettiva

Allora considero la RESTRIZIONE DI $\sin x$ tra $-\frac{\pi}{2}$ e $\frac{\pi}{2}$ (all'intervallo $[-\frac{\pi}{2}, \frac{\pi}{2}]$)

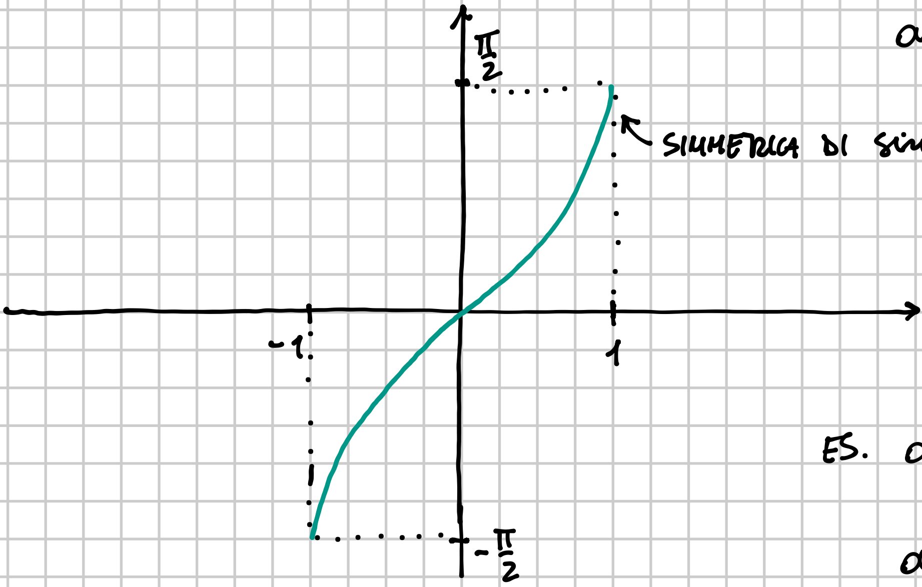


← ora è invertibile

$\sin : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, -1]$

L'inversa è la funzione ARCOSENO

$\arcsin : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



SIMMETRICA DI $\sin x$ rispetto alla bisettrice I-III QUADR. $y = x$

ES. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\arcsin(-1) = -\frac{\pi}{2}$

$\arcsin(0) = 0$

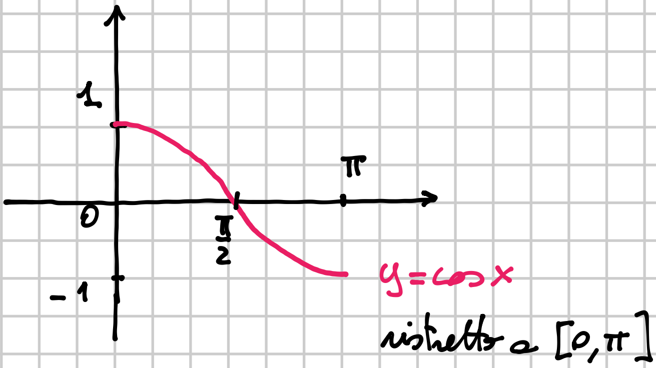
$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

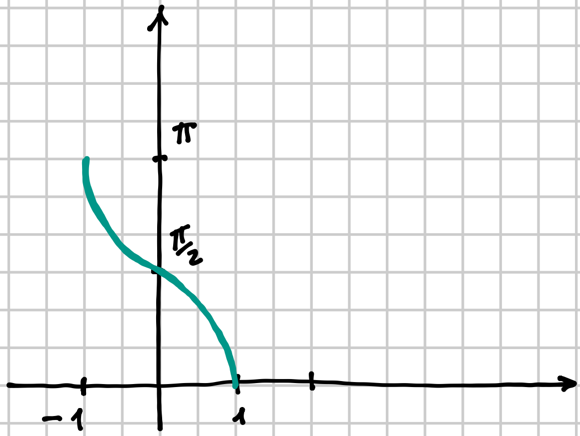
$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

È una funzione DISPARI

$\arcsin(-x) = -\arcsin x$

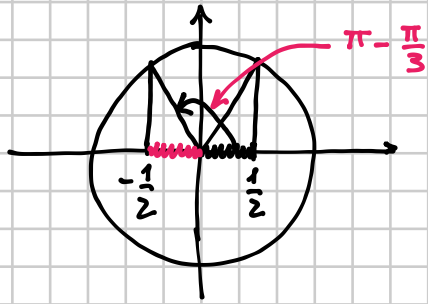
ARCOCOSENO = inversa della restrizione del coseno all'intervallo $[0, \pi]$





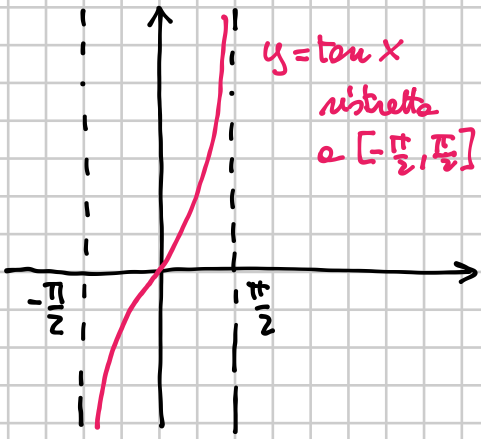
$\cos : [0, \pi] \to [-1, 1]$ $\qquad$ $\arccos : [-1, 1] \to [0, \pi]$

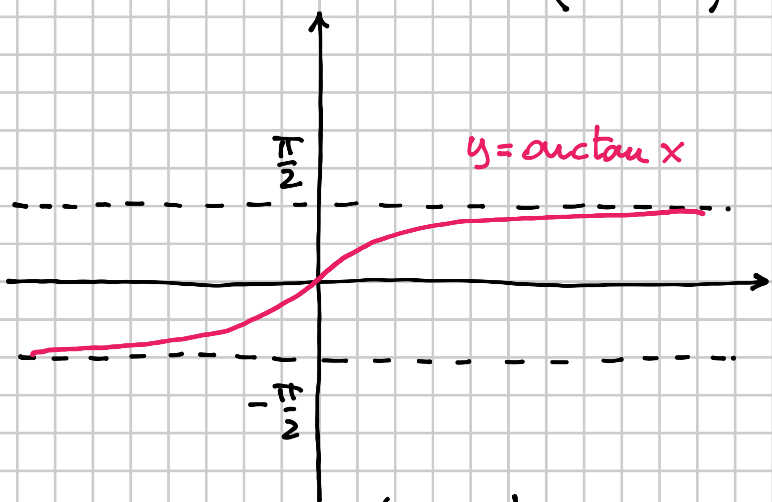
ES. $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\quad$ $\arccos\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} =$



$= \frac{2}{3}\pi$

ARCOTANGENTE = inversa della restrizione della tangente a $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$





$\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ $\qquad$ $\arctan : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

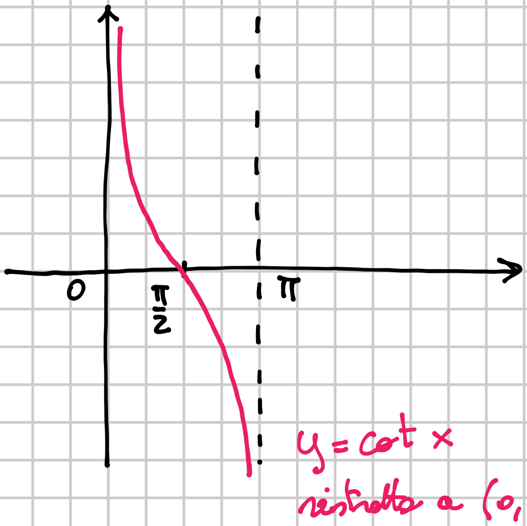
è una funzione dispari $\arctan(-x) = -\arctan x$

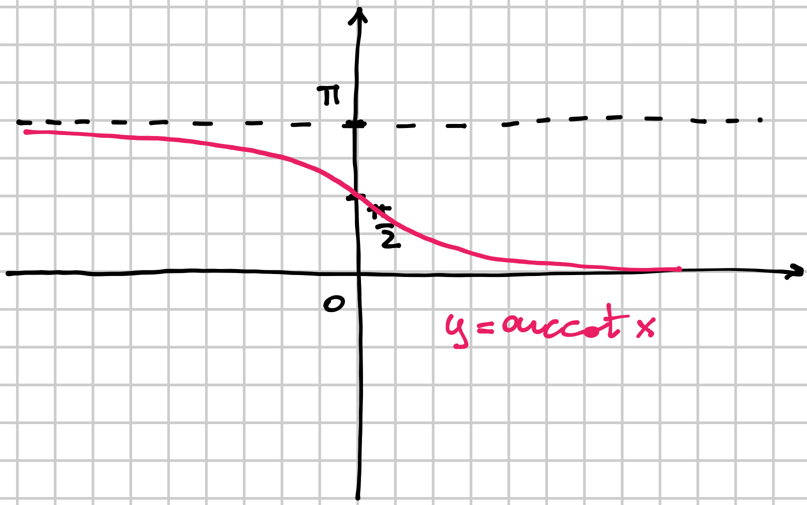
ES. $\arctan(1) = \frac{\pi}{4}$ $\quad$ $\arctan(-1) = -\frac{\pi}{4}$

$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\quad$ $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

$\arctan \sqrt{3} = \frac{\pi}{3}$ $\quad$ $\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

ARCOCOTANGENTE = inversa della cotangente ristretta a $(0, \pi)$





$\cot : (0, \pi) \to \mathbb{R}$ $\qquad$ $\operatorname{arccot} : \mathbb{R} \to (0, \pi)$

499 $\arctan(-1) + 2 \arcsin\frac{1}{2} + \arctan(-\sqrt{3})$ $\left[-\frac{\pi}{4}\right]$

$$= -\frac{\pi}{4} + 2 \cdot \frac{\pi}{6} + \left(-\frac{\pi}{3}\right) = -\frac{\pi}{4} + \cancel{\frac{\pi}{3}} - \cancel{\frac{\pi}{3}} = \boxed{-\frac{\pi}{4}}$$

502 $\arccos(-1) + \overbrace{\arcsin\left(-\frac{1}{2}\right)}^{-\arcsin\left(\frac{1}{2}\right)} - \operatorname{arccot}\sqrt{3} =$ $\left[\frac{2}{3}\pi\right]$

$$= \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{6-2}{6}\pi = \boxed{\frac{2}{3}\pi}$$

521 $\cos\left[\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right] =$ $\left[\frac{1}{2}\right]$

$$= \cos\left[-\arcsin\left(\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \cos\left[-\frac{\pi}{3}\right] = \underbrace{\cos\frac{\pi}{3}}_{\text{se voglio uso gli angoli associati}} = \frac{1}{2}$$

$$\sin\left[\arctan\left(-\frac{4}{3}\right)\right] \stackrel{?}{=} \square,$$

$\arctan\left(-\frac{4}{3}\right)$ è compreso fra $-\frac{\pi}{2}$ e 0, dunque è negativo. Dato che il seno di un angolo tra $-\frac{\pi}{2}$ e 0 è pure negativo, si ha che $\sin\left[\arctan\left(-\frac{4}{3}\right)\right] < 0$. Si poteva anche osservare che

$$\sin\left[\arctan\left(-\frac{4}{3}\right)\right] = \sin\left[-\arctan\left(\frac{4}{3}\right)\right] = -\underbrace{\sin\left[\arctan\frac{4}{3}\right]}_{\text{POSITIVO}}$$

$$\sin\left(\arctan\frac{4}{3}\right) = ?$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} \qquad \tan^2\alpha = \frac{\sin^2\alpha}{1-\sin^2\alpha}$$

$$\left(1-\sin^2\alpha\right)\tan^2\alpha = \sin^2\alpha \qquad \tan^2\alpha - \sin^2\alpha \cdot \tan^2\alpha - \sin^2\alpha = 0$$

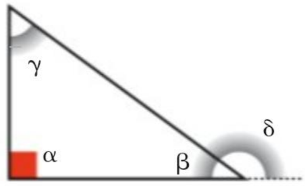
$$\sin^2\alpha\left(-\tan^2\alpha - 1\right) = -\tan^2\alpha \implies \sin^2\alpha = \frac{\tan^2\alpha}{1+\tan^2\alpha}$$

$$\sin\alpha = \pm\sqrt{\frac{\tan^2\alpha}{1+\tan^2\alpha}}$$

$$\sin\left(\arctan\frac{4}{3}\right) = +\sqrt{\frac{\tan^2\left(\arctan\frac{4}{3}\right)}{1+\tan^2\left(\arctan\frac{4}{3}\right)}} = \sqrt{\frac{\left(\frac{4}{3}\right)^2}{1+\left(\frac{4}{3}\right)^2}} =$$

$$= \sqrt{\frac{\frac{16}{9}}{1+\frac{16}{9}}} = \sqrt{\frac{\frac{16}{\cancel{9}}}{\frac{25}{\cancel{9}}}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

46



$\tan\beta = \frac{4}{3}$

Calcola:
$\sin\beta$, $\cos\beta$, $\cos\gamma$, $\tan\delta$.

$\left[\frac{4}{5}; \frac{3}{5}; \frac{4}{5}; -\frac{4}{3}\right]$

$$\tan\delta = \tan(\pi - \beta) =$$

$$= -\tan\beta = -\frac{4}{3}$$

$$\begin{cases} \tan\beta = \dfrac{\sin\beta}{\cos\beta} = \dfrac{4}{3} \\ \sin^2\beta + \cos^2\beta = 1 \end{cases} \quad \begin{cases} \sin\beta = \dfrac{4}{3}\cos\beta \\ \dfrac{16}{9}\cos^2\beta + \cos^2\beta = 1 \end{cases}$$

$$\begin{cases} // \\ \dfrac{25}{9}\cos^2\beta = 1 \end{cases} \quad \begin{cases} // \\ \cos^2\beta = \dfrac{9}{25} \end{cases} \quad \begin{cases} \sin\beta = \dfrac{4}{3}\cdot\dfrac{3}{5} = \dfrac{4}{5} \\ \cos\beta = +\dfrac{3}{5} \end{cases}$$

perché β è un angolo acuto

$$\cos\gamma = \cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta = \frac{4}{5}$$