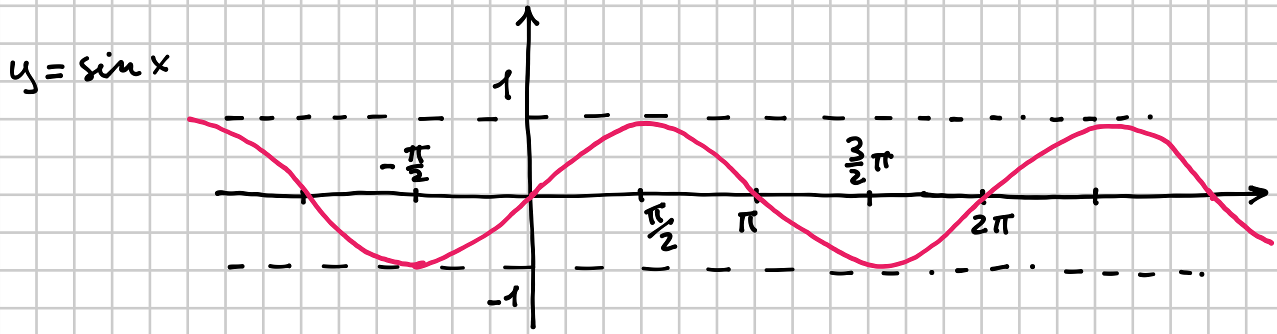
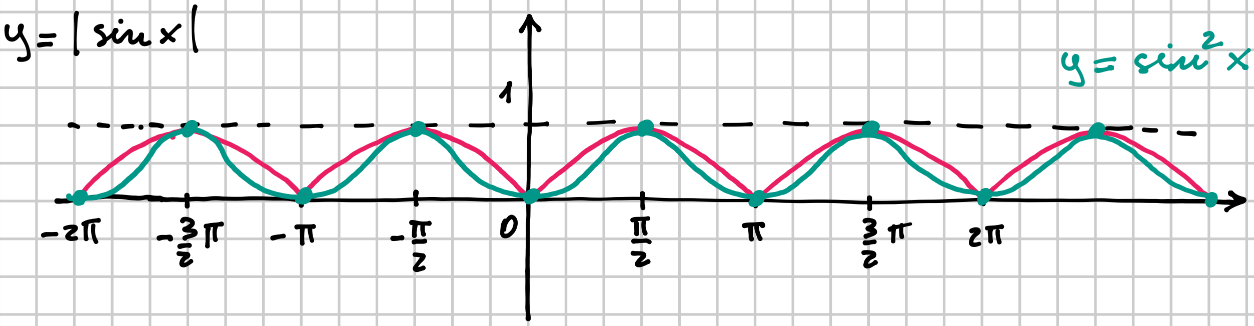
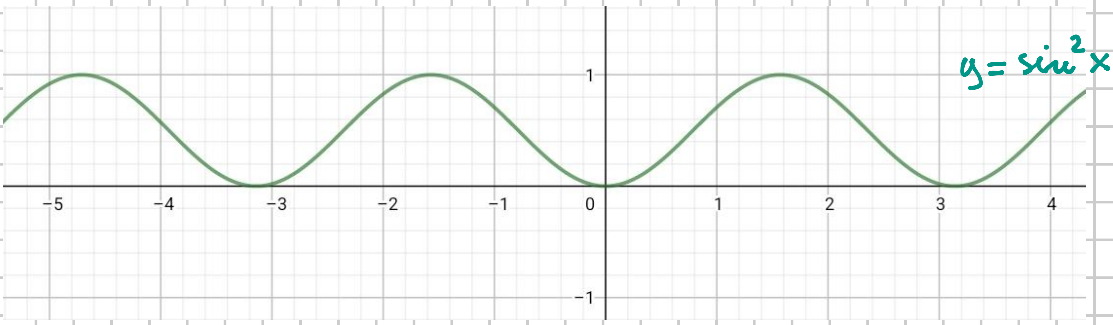
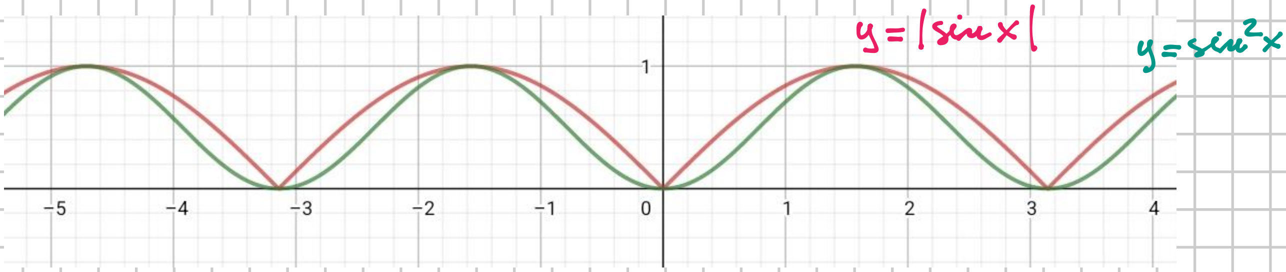
13/10/2022

Disegniamo la funzione $y = \sin^2 x$









$-1 \leq \sin x \leq 1 \implies 0 \leq |\sin x| \leq 1$ quindi

$\sin^2 x = |\sin x|^2$

e $\sin^2 x \leq |\sin x|$

perché il quadrato di un numero fra 0 e 1 è minore del numero stesso

(es. $\left(\frac{1}{2}\right)^2 = \frac{1}{4} < \frac{1}{2}$)

$\arcsin\left(\sin \frac{5}{2}\pi\right)$ NON è $\frac{5}{2}\pi$!!

Infatti $\arcsin(x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin \frac{5}{2}\pi = \sin\left(\cancel{2\pi} + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$$

$$\arcsin\left(\sin \frac{5}{2}\pi\right) = \arcsin(1) = \frac{\pi}{2}$$

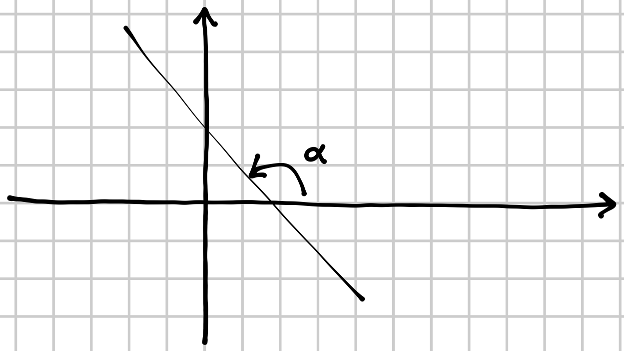
36 $\sin\left(\alpha + \frac{3}{2}\pi\right)\cos(\alpha+\pi) - \dfrac{\tan\left(\frac{3}{2}\pi - \alpha\right)\sin\left(\frac{\pi}{2}+\alpha\right)}{\sin(-\alpha)+\cos\left(\frac{\pi}{2}+\alpha\right)} =$ $\left[\dfrac{\cos^2\alpha(2\sin^2\alpha+1)}{2\sin^2\alpha}\right]$

$$= -\cos\alpha\,(-\cos\alpha) - \frac{\cot\alpha \cos\alpha}{-\sin\alpha - \sin\alpha} =$$

$$= \cos^2\alpha - \frac{\frac{\cos\alpha}{\sin\alpha}\cdot\cos\alpha}{-2\sin\alpha} = \cos^2\alpha + \frac{\cos^2\alpha}{2\sin^2\alpha} =$$

$$= \cos^2\alpha\left(1 + \frac{1}{2\sin^2\alpha}\right) = \cos^2\alpha\,\frac{2\sin^2\alpha+1}{2\sin^2\alpha} = \boxed{\frac{\cos^2\alpha(2\sin^2\alpha+1)}{2\sin^2\alpha}}$$

43 Determina il seno dell'angolo che la retta di equazione $12x + 9y - 1 = 0$ forma con l'asse x. $\left[\frac{4}{5}\right]$



$\tan\alpha$ = coeff. angolare della retta

$$12x + 9y - 1 = 0$$

$$m = -\frac{a}{b} = -\frac{12}{9} = -\frac{4}{3}$$

$$\tan\alpha = -\frac{4}{3} \Rightarrow \alpha = \arctan\left(-\frac{4}{3}\right)$$

$$\sin\alpha = \sin\left(\arctan\left(-\frac{4}{3}\right)\right)$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} \qquad \tan^2\alpha = \frac{\sin^2\alpha}{1-\sin^2\alpha}$$

$$(1-\sin^2\alpha)\tan^2\alpha = \sin^2\alpha \qquad \tan^2\alpha - \sin^2\alpha\tan^2\alpha - \sin^2\alpha = 0$$

$$-\sin^2\alpha\left(\tan^2\alpha + 1\right) = -\tan^2\alpha$$

$$\sin^2\alpha = \frac{\tan^2\alpha}{\tan^2\alpha + 1} \Rightarrow \sin\alpha = \pm\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$

nel nostro caso:

$$\sin\alpha = -\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$

$$\sin\left(\arctan\left(-\frac{4}{3}\right)\right) = -\frac{\tan\left(\arctan\left(-\frac{4}{3}\right)\right)}{\sqrt{1+\tan^2\left(\arctan\left(-\frac{4}{3}\right)\right)}} = -\frac{-\frac{4}{3}}{\sqrt{1+\left(-\frac{4}{3}\right)^2}} =$$

$$= \frac{\frac{4}{3}}{\sqrt{1+\frac{16}{9}}} = \frac{\frac{4}{3}}{\sqrt{\frac{25}{9}}} = \frac{\frac{4}{3}}{\frac{5}{3}} = \boxed{\frac{4}{5}}$$