21/10/2022

61 $\tan\left(\arcsin\frac{3}{5} - \arcsin\frac{1}{2}\right) =$ $\left[\frac{48-25\sqrt{3}}{39}\right]$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{\tan\left(\arcsin\frac{3}{5}\right) - \tan\left(\arcsin\frac{1}{2}\right)}{1 + \tan\left(\arcsin\frac{3}{5}\right)\tan\left(\arcsin\frac{1}{2}\right)} = (*)$$

A PARTE CALCOLO:

$$\tan\left(\arcsin\frac{3}{5}\right) = \frac{\sin\left(\arcsin\frac{3}{5}\right)}{\cos\left(\arcsin\frac{3}{5}\right)} = \frac{\frac{3}{5}}{\sqrt{1-\sin^2\left(\arcsin\frac{3}{5}\right)}} = \frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}} =$$

$$= \frac{\frac{3}{5}}{\sqrt{\frac{16}{25}}} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

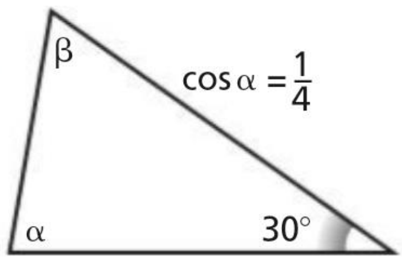
$$\tan\left(\arcsin\frac{1}{2}\right) = \frac{\sin\left(\arcsin\frac{1}{2}\right)}{\cos\left(\arcsin\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\sqrt{1-\underbrace{\frac{1}{4}}_{\left(\frac{1}{2}\right)^2}}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$(*) = \frac{\frac{3}{4} - \frac{\sqrt{3}}{3}}{1 + \frac{\cancel{3}}{4}\cdot\frac{\sqrt{3}}{\cancel{3}}} = \frac{\frac{9-4\sqrt{3}}{12}}{1+\frac{\sqrt{3}}{4}} = \frac{\frac{9-4\sqrt{3}}{12}}{\frac{4+\sqrt{3}}{4}} =$$

$$= \frac{9-4\sqrt{3}}{\cancel{12}_{\,3}}\cdot\frac{\cancel{4}^{\,1}}{4+\sqrt{3}} = \frac{9-4\sqrt{3}}{3(4+\sqrt{3})}\cdot\frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{36-9\sqrt{3}-16\sqrt{3}+12}{3(16-3)}$$

$$= \boxed{\frac{48-25\sqrt{3}}{39}}$$

69



Determina tan β.

$$\left[\frac{\sqrt{3}(4+\sqrt{5})}{3}\right]$$

$$\beta = 180^\circ - \alpha - 30^\circ = 150^\circ - \alpha$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{16}} = \frac{\sqrt{15}}{4}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = \sqrt{15}$$

$$\tan\beta = \tan(150^\circ - \alpha) = \frac{\tan 150^\circ - \tan\alpha}{1+\tan 150^\circ \cdot \tan\alpha} = \frac{-\frac{\sqrt{3}}{3} - \sqrt{15}}{1-\frac{\sqrt{3}}{3}\cdot\sqrt{15}} =$$

$$\tan 150^\circ = \tan(180^\circ - 30^\circ) =$$
$$= \tan(-30^\circ) =$$
$$= -\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

$$= \frac{\frac{-\sqrt{3}-3\sqrt{15}}{3}}{1-\sqrt{5}} =$$

$$= \frac{-\sqrt{3}(1+3\sqrt{5})}{3}\cdot\frac{1}{1-\sqrt{5}} =$$

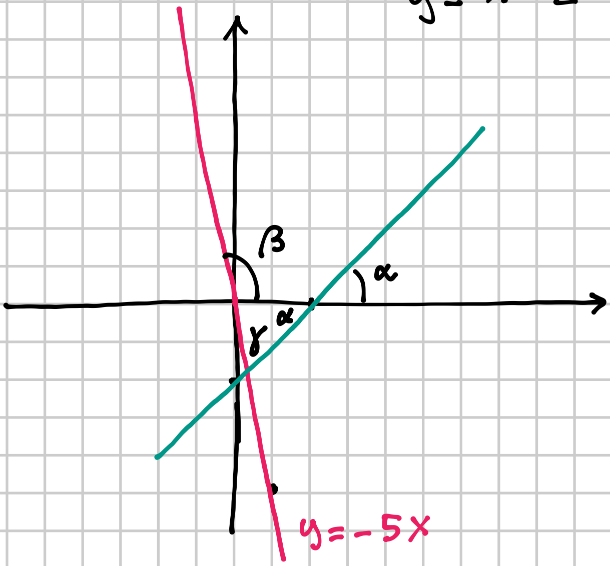
$$= \frac{-\sqrt{3}(1+3\sqrt{5})}{3(1-\sqrt{5})}\cdot\frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{-\sqrt{3}(1+\sqrt{5}+3\sqrt{5}+15)}{3(1-5)} = \frac{-\sqrt{3}(16+4\sqrt{5})}{-12} =$$

$$= \frac{-\cancel{4}\sqrt{3}(4+\sqrt{5})}{-\cancel{12}_{3}} = \boxed{\frac{\sqrt{3}(4+\sqrt{5})}{3}}$$

Calcolare la tangente goniometrica dell'angolo acuto formato dalle due rette

84 $y = -5x$; $\quad x - y = 2$. $\quad \left[\frac{3}{2}\right]$

$y = x - 2$



$\tan\alpha = 1 \qquad \tan\beta = -5$

$\beta = \alpha + \gamma$

$\gamma = \beta - \alpha$

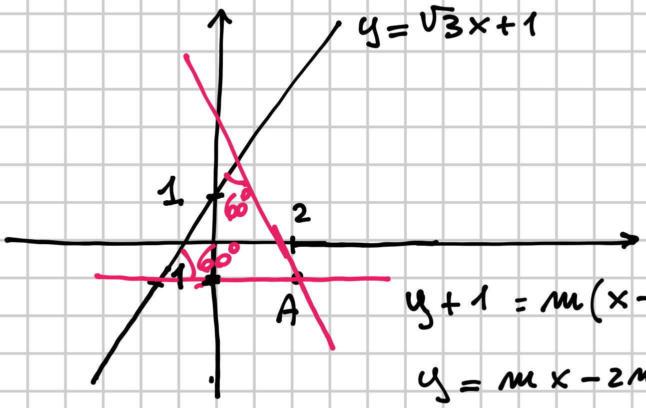
$\tan\gamma = \tan(\beta - \alpha) =$

$= \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha} = \frac{-5-1}{1-5} = \frac{-6}{-4} =$

$= \frac{3}{2}$

93 Trova le equazioni delle rette passanti per il punto $A(2; -1)$ che formano un angolo di 60° con la retta di equazione $y = \sqrt{3}\,x + 1$.

$[y = -1,\ y = -\sqrt{3}\,x + 2\sqrt{3} - 1]$



$\tan 60° = \sqrt{3}$

m DA TROVARE $\qquad m' = \sqrt{3}$

$y + 1 = m(x-2)$

$y = mx - 2m - 1$

$\tan 60° = \pm \frac{m - m'}{1 + mm'}$

$\sqrt{3} = \frac{m - \sqrt{3}}{1 + m\sqrt{3}}$

$\sqrt{3}(1 + m\sqrt{3}) = m - \sqrt{3}$

$\sqrt{3} + 3m = m - \sqrt{3}$

$2m = -2\sqrt{3}$

$m = -\sqrt{3} \Rightarrow \boxed{y = -\sqrt{3}x + 2\sqrt{3} - 1}$

$\sqrt{3} = \frac{\sqrt{3} - m}{1 + m\sqrt{3}}$

$\sqrt{3}(1 + m\sqrt{3}) = \sqrt{3} - m$

$\cancel{\sqrt{3}} + 3m = \cancel{\sqrt{3}} - m$

$-2m = 0$

$m = 0 \Rightarrow \boxed{y = -1}$