26/10/2022

117 $\tan 2\alpha \cdot (1+\tan\alpha)\cdot\cot\alpha$ $\left[\dfrac{2}{1-\tan\alpha}\right]$

$$= \frac{2\cancel{\tan\alpha}}{1-\tan^2\alpha}(1+\tan\alpha)\cdot\frac{1}{\cancel{\tan\alpha}} =$$

$$= \frac{2\cancel{(1+\tan\alpha)}}{(1-\tan\alpha)\cancel{(1+\tan\alpha)}} = \boxed{\frac{2}{1-\tan\alpha}}$$

127 $2\cos\alpha\cdot(1+\cos 2\alpha) - \sin\alpha\sin 2\alpha =$

$$= 2\cos\alpha\left(\cancel{1}+2\cos^2\alpha-\cancel{1}\right) - \sin\alpha\cdot 2\sin\alpha\cos\alpha =$$

$$= 4\cos^3\alpha - 2\sin^2\alpha\cdot\cos\alpha = 4\cos^3\alpha - 2(1-\cos^2\alpha)\cos\alpha =$$

$$= 4\cos^3 x - 2\cos\alpha + 2\cos^3\alpha = 6\cos^3\alpha - 2\cos\alpha =$$

$$= \boxed{2\cos\alpha\,(3\cos^2\alpha - 1)}$$

128 $\sin 2\alpha - 2\sin\alpha(\cos\alpha+1) - \cos 2\alpha + 1 - 2\sin\alpha(\sin\alpha - 1) =$

$$= \cancel{2\sin\alpha\cos\alpha} - \cancel{2\sin\alpha\cos\alpha} - \cancel{2\sin\alpha} - \cos 2\alpha + 1 - 2\sin^2\alpha + \cancel{2\sin\alpha}$$

$$= -(1-2\sin^2\alpha) + 1 - 2\sin^2\alpha =$$

$$= -\cancel{1} + \cancel{2\sin^2\alpha} + \cancel{1} - \cancel{2\sin^2\alpha} = \boxed{0}$$

124 $\sin\left(2\alpha - \frac{\pi}{6}\right) + 2\cos^2\left(\frac{\pi}{3} + \alpha\right) =$ $[2\sin^2\alpha]$

$$= \sin 2\alpha \cdot \cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos 2\alpha + 2\left[\cos\frac{\pi}{3}\cdot\cos\alpha - \sin\frac{\pi}{3}\cdot\sin\alpha\right]^2 =$$

$$= \cancel{2}\sin\alpha\cos\alpha\cdot\frac{\sqrt{3}}{\cancel{2}} - \frac{1}{2}\cos 2\alpha + 2\left[\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha\right]^2 =$$

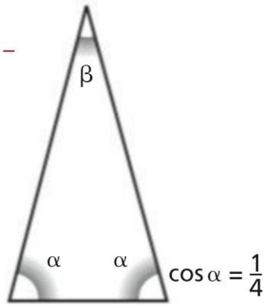
$$= \sqrt{3}\sin\alpha\cos\alpha - \frac{1}{2}\cos 2\alpha + 2\left[\frac{1}{4}\cos^2\alpha + \frac{3}{4}\sin^2\alpha - \frac{\sqrt{3}}{2}\sin\alpha\cos\alpha\right] =$$

$$= \cancel{\sqrt{3}\sin\alpha\cos\alpha} - \frac{1}{2}\cos 2\alpha + \frac{1}{2}\cos^2\alpha + \frac{3}{2}\sin^2\alpha - \cancel{\sqrt{3}\sin\alpha\cos\alpha} =$$

$$= -\frac{1}{2}\left(\cos^2\alpha - \sin^2\alpha\right) + \frac{1}{2}\cos^2\alpha + \frac{3}{2}\sin^2\alpha =$$

$$= \cancel{-\frac{1}{2}\cos^2\alpha} + \frac{1}{2}\sin^2\alpha + \cancel{\frac{1}{2}\cos^2\alpha} + \frac{3}{2}\sin^2\alpha = \boxed{2\sin^2\alpha}$$

164



Calcola sin β e cos β. $\left[\frac{\sqrt{15}}{8}; \frac{7}{8}\right]$

$$\beta = \pi - 2\alpha$$

$$\sin\beta = \sin(\pi - 2\alpha) = \sin 2\alpha =$$

$$= 2\sin\alpha\cos\alpha$$

$$\sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{16}} = \frac{\sqrt{15}}{4}$$

$$= \cancel{2}\cdot\frac{\sqrt{15}}{\cancel{4}_{2}}\cdot\frac{1}{4} = \boxed{\frac{\sqrt{15}}{8}}$$

$$\cos\beta = \cos(\pi - 2\alpha) = -\cos 2\alpha =$$

$$= -(2\cos^2\alpha - 1) =$$

$$= 1 - 2\cos^2\alpha = 1 - 2\cdot\frac{1}{16} =$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

CALCOLARE SAPENDO CHE

148 $\sin 2\beta$, $\cos 2\beta$; $\cot\beta = -3$, con $\frac{3}{2}\pi < \beta < 2\pi$

$\hookrightarrow \cos\beta > 0$

$\sin\beta < 0$

$$\begin{cases} \dfrac{\cos\beta}{\sin\beta} = -3 \\ \sin^2\beta + \cos^2\beta = 1 \end{cases} \qquad \begin{cases} \cos\beta = -3\sin\beta \\ \sin^2\beta + 9\sin^2\beta = 1 \end{cases} \qquad \begin{cases} // \\ 10\sin^2\beta = 1 \end{cases}$$

$$\begin{cases} // \\ \sin^2\beta = \dfrac{1}{10} \end{cases} \qquad \begin{cases} \cos\beta = -3\left(-\dfrac{1}{\sqrt{10}}\right) = \dfrac{3}{\sqrt{10}} \\ \sin\beta = -\dfrac{1}{\sqrt{10}} \end{cases}$$

MENO perché β è nel IV QUADR.

$$\sin 2\beta = 2\sin\beta\cos\beta = 2\cdot\left(-\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) = -\frac{6}{10} = \boxed{-\frac{3}{5}}$$

$$\cos 2\beta = 2\cos^2\beta - 1 = 2\left(\frac{3}{\sqrt{10}}\right)^2 - 1 = \cancel{2}\cdot\frac{9}{\cancel{10}_{5}} - 1 = \boxed{\frac{4}{5}}$$

DISEGNARE

189 $y = \sin x \cos x + \cos^2 x$

$$\cos 2x = 2\cos^2 x - 1$$
$$2\cos^2 x = 1 + \cos 2x$$
$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$y = \frac{1}{2}\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}$$

$$y = \frac{1}{2}(\sin 2x + \cos 2x) + \frac{1}{2}$$

$2x = t$

$$\sin t + \cos t = r \cdot \sin(t + \alpha) \qquad r > 0$$
$$= r[\sin t \cos\alpha + \cos t \sin\alpha] =$$
$$= r\cos\alpha \cdot \sin t + r\sin\alpha \cdot \cos t$$

$$\begin{cases} r\cos\alpha = 1 \\ r\sin\alpha = 1 \end{cases}$$

$$r^2\cos^2\alpha + r^2\sin^2\alpha = 2$$
$$r^2\underbrace{(\cos^2\alpha + \sin^2\alpha)}_{1} = 2 \Rightarrow r = \sqrt{2}$$

$$\frac{r\sin\alpha}{r\cos\alpha} = 1$$
$$\Downarrow$$
$$\tan\alpha = 1 \qquad \alpha = \frac{\pi}{4}$$

perché $\cos\alpha > 0$
$\sin\alpha > 0$

$$\sin t + \cos t = \sqrt{2}\sin\left(t + \frac{\pi}{4}\right)$$

Quindi la funzione di partenza è

$$y = \frac{1}{2}\left(\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)\right) + \frac{1}{2}$$

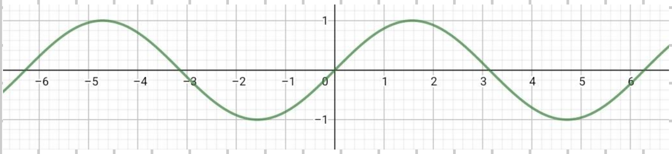
$$y = \frac{\sqrt{2}}{2}\sin\left(2x + \frac{\pi}{4}\right) + \frac{1}{2}$$

Si può disegnare con le trasformazioni elementari

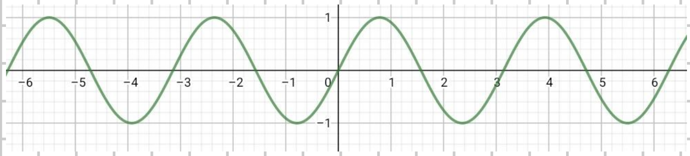
$$y = \frac{\sqrt{2}}{2} \sin\left(2x + \frac{\pi}{4}\right) + \frac{1}{2}$$

PASSI

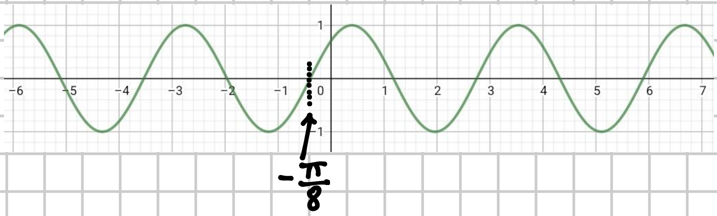
$$y = \sin x$$



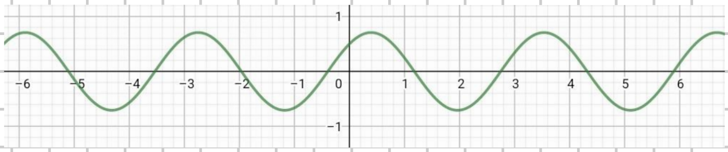
$$y = \sin 2x$$



$$y = \sin\left(2\left(x + \frac{\pi}{8}\right)\right) = \sin\left(2x + \frac{\pi}{4}\right)$$



$$y = \frac{\sqrt{2}}{2} \sin\left(2x + \frac{\pi}{4}\right)$$



$$y = \frac{\sqrt{2}}{2} \sin\left(2x + \frac{\pi}{4}\right) + \frac{1}{2}$$