DISEGNARE

28/10/2022

193 $y = \sqrt{3}\sin^2 x + \sin x \cos x - 1$

$\sin x \cos x = \frac{1}{2}\sin 2x$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$y = \sqrt{3}\,\frac{1-\cos 2x}{2} + \frac{1}{2}\sin 2x - 1$$

$$y = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x - 1$$

$$y = -\frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x + \frac{\sqrt{3}-2}{2}$$

$2x = t$

$$-\frac{\sqrt{3}}{2}\cos t + \frac{1}{2}\sin t = r\sin(t+\alpha)$$

$r > 0$

r, α da trovare

$$= r\sin t\cdot\cos\alpha + r\cos t\cdot\sin\alpha =$$

$$= r\sin\alpha\cos t + r\cos\alpha\sin t$$

$$\begin{cases} r\sin\alpha = -\frac{\sqrt{3}}{2} \\ r\cos\alpha = \frac{1}{2} \end{cases}$$

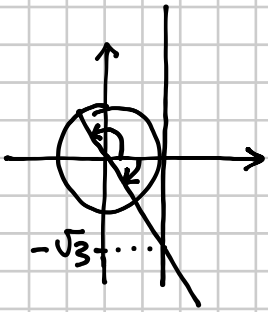
$$r^2\sin^2\alpha + r^2\cos^2\alpha = \frac{3}{4} + \frac{1}{4}$$

$$r^2\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1} = 1 \implies \boxed{r = 1}$$

$$\frac{\cancel{r}\sin\alpha}{\cancel{r}\cos\alpha} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$\Downarrow$

$\tan\alpha = -\sqrt{3}$



$\boxed{\alpha = -\frac{\pi}{3}}$ $\vee$ $\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2}{3}\pi$

$\downarrow$ perché $\sin\alpha < 0$ e $\cos\alpha > 0$

Quindi $-\frac{\sqrt{3}}{2}\cos t + \frac{1}{2}\sin t = \sin\left(t - \frac{\pi}{3}\right)$, da cui:

$$y = \sin\left(2x - \frac{\pi}{3}\right) + \frac{\sqrt{3}-2}{2}$$

$$y = \sin\left(2x - \frac{\pi}{3}\right) + \frac{\sqrt{3}-2}{2}$$

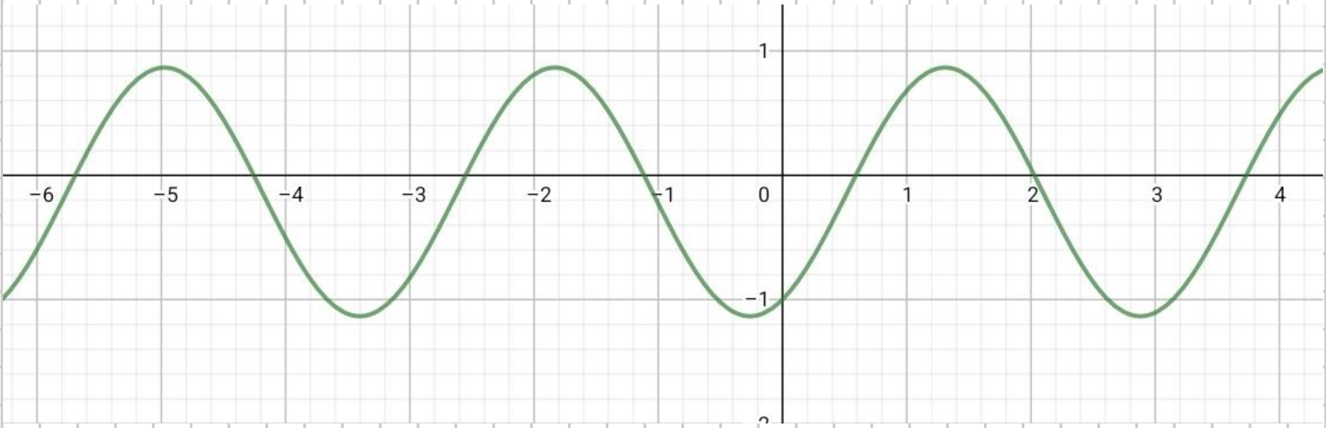
PASSI

$y = \sin x$

$y = \sin 2x$

$y = \sin\left(2\left(x - \frac{\pi}{6}\right)\right) = \sin\left(2x - \frac{\pi}{3}\right)$

$y = \sin\left(2x - \frac{\pi}{3}\right) + \frac{\sqrt{3}-2}{2}$



140 $$\frac{\cos 2\alpha}{\cos\alpha-\sin\alpha}-\frac{\sin 2\alpha}{\cos\alpha+\sin\alpha}=\frac{1}{\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)}$$

$$\frac{(\cos\alpha+\sin\alpha)\cos 2\alpha-(\cos\alpha-\sin\alpha)\overbrace{\sin 2\alpha}^{2\sin\alpha\cos\alpha}}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}=\frac{1}{\sqrt{2}\left[\sin\alpha\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}+\cos\alpha\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\right]}$$

$$\frac{(\cos\alpha+\sin\alpha)\cos 2\alpha-2\sin\alpha\cos^2\alpha+2\sin^2\alpha\cos\alpha}{(\cos\alpha-\sin\alpha)\cancel{(\cos\alpha+\sin\alpha)}}=\frac{1}{\cancel{\sin\alpha+\cos\alpha}}$$

$$\frac{(\cos\alpha+\sin\alpha)(1-2\sin^2\alpha)-2\sin\alpha\cos^2\alpha+2\sin^2\alpha\cos\alpha}{\cos\alpha-\sin\alpha}=1$$

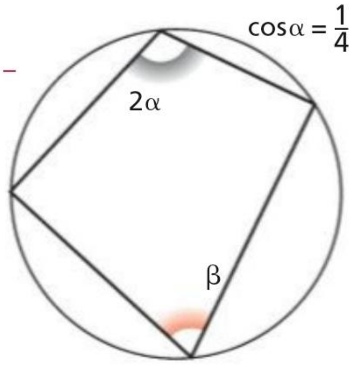
$$\frac{\cos\alpha-\cancel{2\sin^2\alpha\cos\alpha}+\sin\alpha-2\sin^3\alpha-2\sin\alpha\cos^2\alpha+\cancel{2\sin^2\alpha\cos\alpha}}{\cos\alpha-\sin\alpha}=1$$

$$\frac{\cos\alpha+\sin\alpha-2\sin\alpha\overbrace{(\sin^2\alpha+\cos^2\alpha)}^{1}}{\cos\alpha-\sin\alpha}=1$$

$$\frac{\cos\alpha-\sin\alpha}{\cos\alpha-\sin\alpha}=1$$

$1=1$ ok!

165



Determina $\tan\beta$.

$\left[\frac{\sqrt{15}}{7}\right]$

$2\alpha + \beta = \pi$ perché un quadrilatero inscritto in una circonferenza ha gli angoli opposti (interni) supplementari

$$\beta = \pi - 2\alpha$$

$$\tan\beta = \tan(\pi - 2\alpha) = \tan(-2\alpha) = -\tan 2\alpha =$$

$$= -\frac{2\tan\alpha}{1-\tan^2\alpha} =$$

$$\cos\alpha = \frac{1}{4} \qquad (\alpha \text{ ACUTO})$$

$$\sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{16}} =$$

$$= \frac{\sqrt{15}}{4}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \sqrt{15}$$

$$= -\frac{2\sqrt{15}}{1-15} =$$

$$= -\frac{2\sqrt{15}}{-14} = \boxed{\frac{\sqrt{15}}{7}}$$