2/11/2022

194 $y = \sin x \cos x + \frac{\sqrt{3}}{2}(\cos^2 x - \sin^2 x)$ DISEGNARE

$$y = \frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x = r\sin(2x+\alpha) \qquad r>0$$

ANGOLO AGGIUNTO

$$= r\sin 2x\cos\alpha + r\cos 2x\cdot\sin\alpha =$$

$$= r\cos\alpha\cdot\sin 2x + r\sin\alpha\cdot\cos 2x$$

$$\begin{cases} r\sin\alpha = \frac{\sqrt{3}}{2} \\ r\cos\alpha = \frac{1}{2} \end{cases}$$

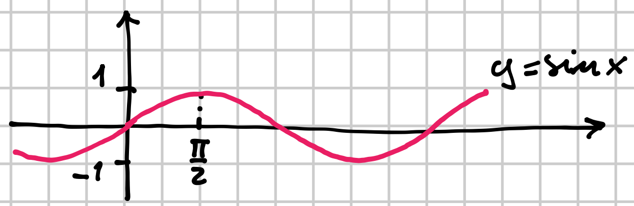
$$\frac{\cancel{r}\sin\alpha}{\cancel{r}\cos\alpha} = \sqrt{3} \Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$r^2\sin^2\alpha + r^2\cos^2\alpha = \frac{3}{4} + \frac{1}{4}$$

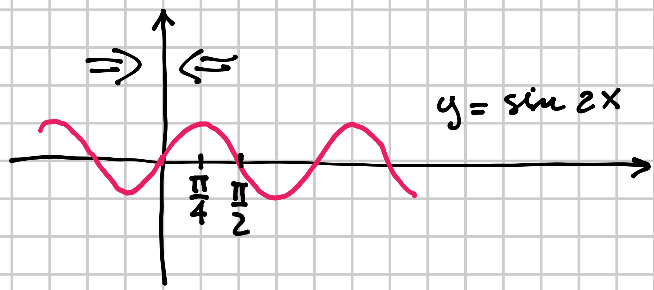
$$r^2\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1} = 1 \Rightarrow r = 1$$

La funzione di partenza è $y = \sin\left(2x + \frac{\pi}{3}\right)$

1) $y = \sin x$

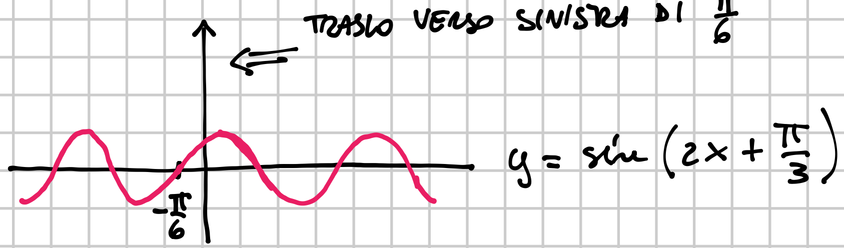


2) $y = \sin 2x$



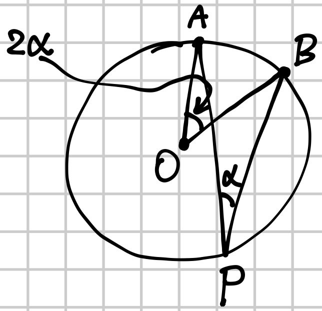
3) $y = \sin\left(2\left(x + \frac{\pi}{6}\right)\right)$

$= \sin\left(2x + \frac{\pi}{3}\right)$



166 Un angolo alla circonferenza ha ampiezza α e $\cos\alpha = \frac{8}{17}$. Trova seno e coseno del corrispondente angolo al centro.

$\left[\sin\beta = \frac{240}{289};\ \cos\beta = -\frac{161}{289}\right]$



$$A\hat{O}B = 2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 2\cdot\left(\frac{8}{17}\right)^2 - 1 =$$

$$= 2\cdot\frac{64}{289} - 1 = \frac{128-289}{289} = \boxed{-\frac{161}{289}}$$

$\cos\alpha = \frac{8}{17} > 0$ $\quad \sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{289-64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$

è + perché α è
compreso fra 0 e π
(essendo angolo alla circonferenza)

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{15}{17}\cdot\frac{8}{17} = \boxed{\frac{240}{289}}$$

DISEGNARE

196 $y = \frac{1}{2}\sin x \cos x - \frac{1}{2}\cos^2 x$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$y = \frac{1}{4}\sin 2x - \frac{1}{4}\cos 2x - \frac{1}{4}$$

$$y = \frac{1}{4}\left(\sin 2x - \cos 2x\right) - \frac{1}{4}$$

$$r > 0$$

$$\sin 2x - \cos 2x = r \sin(2x + \alpha) = r\cos\alpha \sin 2x + r\sin\alpha\cos 2x$$

α è nel IV quadr.

$$\begin{cases} r\sin\alpha = -1 \\ r\cos\alpha = 1 \end{cases}$$

$$r^2\sin^2\alpha + r^2\cos^2\alpha = 2$$

$$\tan\alpha = -1 \Rightarrow \alpha = -\frac{\pi}{4}$$

$$r^2 = 2 \Rightarrow r = \sqrt{2}$$

$$\sin 2x - \cos 2x = \sqrt{2}\cdot\sin\left(2x - \frac{\pi}{4}\right)$$

$$y = \frac{1}{4}\left(\sqrt{2}\sin\left(2x - \frac{\pi}{4}\right)\right) - \frac{1}{4}$$

$$y = \frac{\sqrt{2}}{4}\sin\left(2x - \frac{\pi}{4}\right) - \frac{1}{4}$$

per disegnarlo i passi sono:

$y = \sin x$

$y = \sin 2x$

$y = \sin 2\left(x - \frac{\pi}{8}\right) = \sin\left(2x - \frac{\pi}{4}\right)$

$y = \frac{\sqrt{2}}{4}\sin\left(2x - \frac{\pi}{4}\right)$

$y = \frac{\sqrt{2}}{4}\sin\left(2x - \frac{\pi}{4}\right) - \frac{1}{4}$

ALTERNATIVO

$y = \sin x$

$y = \sin\left(x - \frac{\pi}{4}\right)$

$y = \sin\left(2x - \frac{\pi}{4}\right)$

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