3/11/2022

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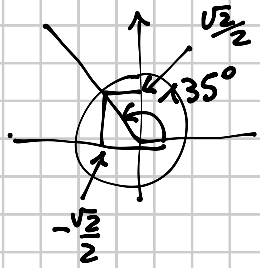
$\cos\alpha = \frac{4}{5}$

Determina sin β e cos β. $\left[\frac{31\sqrt{2}}{50}; \frac{17\sqrt{2}}{50}\right]$

$\beta = 135° - 2\alpha$

$\cos\alpha = \frac{4}{5}$



$\sin\alpha = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$$\sin\beta = \sin(135° - 2\alpha) = \sin 135° \cdot \cos 2\alpha - \cos 135° \cdot \sin 2\alpha =$$

$$= \frac{\sqrt{2}}{2}\cos 2\alpha - \left(-\frac{\sqrt{2}}{2}\right)\sin 2\alpha = \frac{\sqrt{2}}{2}(2\cos^2\alpha - 1) + \frac{\sqrt{2}}{\cancel{2}} \cdot \cancel{2}\sin\alpha\cos\alpha =$$

$$= \frac{\sqrt{2}}{2}\left(2\cdot\frac{16}{25} - 1\right) + \sqrt{2}\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{\sqrt{2}}{2}\,\frac{32-25}{25} + \frac{12\sqrt{2}}{25} =$$

$$= \frac{7\sqrt{2}}{50} + \frac{12\sqrt{2}}{25} = \frac{7\sqrt{2}+24\sqrt{2}}{50} = \boxed{\frac{31\sqrt{2}}{50}}$$

$$\cos\beta = \sqrt{1-\left(\frac{31\sqrt{2}}{50}\right)^2} = \sqrt{1-\frac{1922}{2500}} = \sqrt{\frac{578}{2500}} = \sqrt{\frac{17^2\cdot 2}{2500}} =$$

+ perché β è acuto

$$= \boxed{\frac{17\sqrt{2}}{50}}$$

FORMULE DI BISEZIONE

$\sin\frac{\alpha}{2} = ?$ $\cos\frac{\alpha}{2} = ?$ $\tan\frac{\alpha}{2} = ?$ a partire da $\sin\alpha$
$\cos\alpha$

$\sin\frac{\alpha}{2}$ lo trovo osservando che $\cos 2\alpha = 1 - 2\sin^2\alpha$

$$\Downarrow$$

$$2\sin^2\alpha = 1 - \cos 2\alpha$$

$$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}}$$

$2\alpha = \beta$

$\Downarrow$

$\alpha = \frac{\beta}{2}$

$$\Downarrow$$

$$\sin\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}}$$

Allo stesso modo $\cos\frac{\alpha}{2}$ ... $\cos 2\alpha = 2\cos^2\alpha - 1$

....

$$\cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}}$$ $\alpha = \frac{\beta}{2}$

....

$$\cos\frac{\beta}{2} = \pm\sqrt{\frac{1+\cos\beta}{2}}$$

$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ $\frac{\alpha}{2} \neq \frac{\pi}{2} + K\pi \Rightarrow \alpha \neq \pi + 2K\pi$

IN DEFINITIVA

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad \alpha \neq \pi + 2k\pi$$

Ci sono altri modi di esprimere $\tan\frac{\alpha}{2}$:

1) $$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \implies \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

uso la formula $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

con la condizione $\alpha \neq \pi + 2k\pi$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{\frac{\cancel{1}+\cos\alpha-\cancel{1}+\cos\alpha}{1+\cos\alpha}}$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{2\tan\frac{\alpha}{2}}{\frac{2\cos\alpha}{1+\cos\alpha}}$$

$$\frac{\cancel{2}\cancel{\cos\alpha}}{1+\cos\alpha}\cdot\frac{\sin\alpha}{\cancel{\cos\alpha}} = \cancel{2}\tan\frac{\alpha}{2} \implies \boxed{\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} \qquad \alpha \neq \pi + 2k\pi}$$

2) $$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}\cdot\frac{1-\cos\alpha}{1-\cos\alpha} = \frac{\sin\alpha(1-\cos\alpha)}{1-\cos^2\alpha} = \frac{\cancel{\sin\alpha}(1-\cos\alpha)}{\sin^{\cancel{2}}\alpha} =$$

$\alpha \neq \pi + 2k\pi$ $\qquad$ $\alpha \neq 2k\pi$

$$= \frac{1-\cos\alpha}{\sin\alpha}$$

$\alpha \neq k\pi$

$$\boxed{\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} \qquad \alpha \neq k\pi}$$