4/11/2022

206 $\left(\cos^2\frac{\alpha}{2}-\frac{1}{2}\right)\left(\sin^2\frac{\alpha}{2}-\frac{1}{2}\right) =$ $\left[-\frac{1}{4}\cos^2\alpha\right]$

$$= \left(\frac{1+\cos\alpha}{2}-\frac{1}{2}\right)\left(\frac{1-\cos\alpha}{2}-\frac{1}{2}\right) =$$

$$= \frac{\cancel{1}+\cos\alpha-\cancel{1}}{2}\cdot\frac{\cancel{1}-\cos\alpha-\cancel{1}}{2} = \boxed{-\frac{1}{4}\cos^2\alpha}$$

215 $\tan\frac{\alpha}{2}+\cot\alpha-\csc\alpha+2\sin\alpha =$ $[2\sin\alpha]$

$$= \tan\frac{\alpha}{2}+\frac{\cos\alpha}{\sin\alpha}-\frac{1}{\sin\alpha}+2\sin\alpha =$$

OPPOSTI

$$= \cancel{\frac{1-\cos\alpha}{\sin\alpha}}+\cancel{\frac{\cos\alpha-1}{\sin\alpha}}+2\sin\alpha = \boxed{2\sin\alpha}$$

uso $\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

221 $\cot^2\frac{\alpha}{2}=4\cot\alpha\cdot\csc\alpha+\tan^2\frac{\alpha}{2}$ IDENTITÀ

$$\frac{1}{\tan^2\frac{\alpha}{2}} = 4\cdot\frac{\cos\alpha}{\sin\alpha}\cdot\frac{1}{\sin\alpha}+\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha}{\sin^2\alpha}+\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha}{1-\cos^2\alpha}+\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}+\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)} + \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha + (1-\cos\alpha)^2}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha + 1 + \cos^2\alpha - 2\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{1 + \cos^2\alpha + 2\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{(1+\cos\alpha)^{\cancel{2}}}{(1-\cos\alpha)\cancel{(1+\cos\alpha)}}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{1-\cos\alpha} \qquad \text{ok!}$$

198 $\sin\frac{\pi}{8}$; $\tan\frac{\pi}{8}$. $\left[\frac{1}{2}\sqrt{2-\sqrt{2}};\ \sqrt{2}-1\right]$

$$\sin\frac{\pi}{8} = \sin\left(\frac{\frac{\pi}{4}}{2}\right) = +\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} =$$

$\frac{\pi}{8}$ ACUTO

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

$$\tan\frac{\pi}{8} = \tan\left(\frac{\frac{\pi}{4}}{2}\right) = \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{2+\sqrt{2}}{\cancel{2}}} = \frac{\sqrt{2}}{2+\sqrt{2}} =$$

$$= \frac{\sqrt{2}}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{2\sqrt{2}-2}{4-2} = \frac{\cancel{2}(\sqrt{2}-1)}{\cancel{2}} = \boxed{\sqrt{2}-1}$$

235 $\sin\left(\frac{\alpha}{2}+\beta\right)$; $\tan\alpha = \frac{8}{15}$, $\sin\beta = \frac{3}{5}$, con $0 < \alpha < \frac{\pi}{2}$, $0 < \beta < \frac{\pi}{2}$. $\left[\frac{16\sqrt{17}}{85}\right]$

$$\sin\left(\frac{\alpha}{2}+\beta\right) = \sin\frac{\alpha}{2}\cos\beta + \cos\frac{\alpha}{2}\sin\beta = (*)$$

$$\begin{cases} \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{8}{15} \\ \sin^2\alpha + \cos^2\alpha = 1 \end{cases} \qquad \begin{cases} \sin\alpha = \dfrac{8}{15}\cos\alpha \\ \dfrac{64}{225}\cos^2\alpha + \cos^2\alpha = 1 \end{cases} \qquad \begin{cases} // \\ \dfrac{289}{225}\cos^2\alpha = 1 \end{cases}$$

$$\begin{cases} // \\ \cos^2\alpha = \dfrac{225}{289} \end{cases} \qquad \begin{cases} \sin\alpha = \dfrac{8}{\cancel{15}}\cdot\dfrac{\cancel{15}}{17} = \dfrac{8}{17} \\ \cos\alpha = \dfrac{15}{17} \end{cases}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{15}{17}}{2}} = \sqrt{\frac{\frac{2}{17}}{2}} = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{15}{17}}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$(*) = \frac{\sqrt{17}}{17}\cdot\frac{4}{5} + \frac{4\sqrt{17}}{17}\cdot\frac{3}{5} = \frac{4\sqrt{17}+12\sqrt{17}}{85} = \boxed{\frac{16\sqrt{17}}{85}}$$