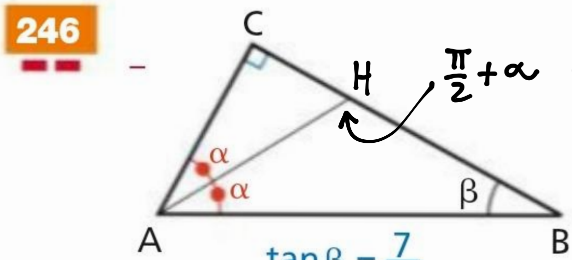
7/11/2022

246



$\tan\beta = \frac{7}{24}$

$\frac{\pi}{2}+\alpha$ (angolo esterno di AHC, somma degli altri 2 angoli interni non adiacenti)

β ACUTO

$\left.\begin{array}{l}\sin\beta\\ \cos\beta\\ \tan\beta\end{array}\right\} > 0$

$$\alpha+\beta+\left(\frac{\pi}{2}+\alpha\right)=\pi$$

somma degli angoli interni di ABH

Calcola $\sin\alpha$ e $\cos\alpha$. $\left[\frac{3}{5};\frac{4}{5}\right]$

$$2\alpha+\beta+\frac{\pi}{2}=\pi$$

$$2\alpha+\beta=\frac{\pi}{2}$$

$$2\alpha=\frac{\pi}{2}-\beta$$

$$\alpha=\frac{\pi}{4}-\frac{\beta}{2}$$

$$\sin\alpha=\sin\left(\frac{\pi}{4}-\frac{\beta}{2}\right)=$$

$$=\sin\frac{\pi}{4}\cos\frac{\beta}{2}-\cos\frac{\pi}{4}\sin\frac{\beta}{2}=$$

$$=\frac{\sqrt{2}}{2}\cos\frac{\beta}{2}-\frac{\sqrt{2}}{2}\sin\frac{\beta}{2}=(*)$$

$$\tan\beta=\frac{7}{24}\Rightarrow\begin{cases}\frac{\sin\beta}{\cos\beta}=\frac{7}{24}\\ \sin^2\beta+\cos^2\beta=1\end{cases}\qquad\begin{cases}\sin\beta=\frac{7}{24}\cos\beta\\ \frac{49}{576}\cos^2\beta+\cos^2\beta=1\end{cases}$$

$$\begin{cases}//\\ \frac{625}{576}\cos^2\beta=1\end{cases}\qquad\begin{cases}\sin\beta=\frac{7}{24}\cdot\frac{24}{25}=\frac{7}{25}\\ \cos\beta=\frac{24}{25}\end{cases}$$

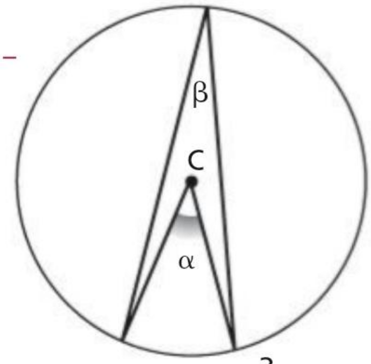
$$(*)=\frac{\sqrt{2}}{2}\sqrt{\frac{1+\cos\beta}{2}}-\frac{\sqrt{2}}{2}\sqrt{\frac{1-\cos\beta}{2}}=\frac{\sqrt{2}}{2}\sqrt{\frac{49}{50}}-\frac{\sqrt{2}}{2}\sqrt{\frac{1}{50}}=$$

$$=\frac{\cancel{\sqrt{2}}}{2}\frac{7}{5\cancel{\sqrt{2}}}-\frac{\cancel{\sqrt{2}}}{2}\cdot\frac{1}{5\cancel{\sqrt{2}}}=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\boxed{\frac{3}{5}}$$

$$\cos\alpha=+\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\boxed{\frac{4}{5}}$$

α ACUTO

245



Calcola sinβ e cosβ. $\left[\frac{\sqrt{10}}{10}; \frac{3\sqrt{10}}{10}\right]$

$$\alpha = 2\beta \Rightarrow \beta = \frac{\alpha}{2}$$

$$\begin{cases} \sin\alpha = \frac{3}{4}\cos\alpha & \left(\tan\alpha = \frac{3}{4}\right) \\ \sin^2\alpha + \cos^2\alpha = 1 \end{cases}$$

$$\frac{9}{16}\cos^2\alpha + \cos^2\alpha = 1$$

$$\cos^2\alpha = \frac{16}{25} \Rightarrow \cos\alpha = \frac{4}{5}$$

(α ACUTO)

$$\sin\beta = \sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} =$$

$$= \sqrt{\frac{1}{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

$$\cos\beta = \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \tan x = \frac{2t}{1-t^2}$$

$t = \tan \frac{x}{2}$ $\qquad x \neq \pi + 2k\pi$ (perché $\frac{x}{2} \neq \frac{\pi}{2} + k\pi$, altrimenti la $\tan \frac{x}{2}$ non esiste!)

DIMOSTRAZIONI

1) $$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{\underbrace{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}}_{1}} = \frac{\frac{2 \sin \frac{x}{2} \cancel{\cos \frac{x}{2}}}{\cos^{\cancel{2}} \frac{x}{2}}}{\frac{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} =$$

$$= \frac{2 \tan \frac{x}{2}}{\frac{\cancel{\cos^2 \frac{x}{2}}}{\cancel{\cos^2 \frac{x}{2}}} + \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2t}{1+t^2} \quad \text{con } t = \tan \frac{x}{2}$$

2) $$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \frac{\frac{\cancel{\cos^2 \frac{x}{2}}}{\cancel{\cos^2 \frac{x}{2}}} - \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}}{\frac{\cancel{\cos^2 \frac{x}{2}}}{\cancel{\cos^2 \frac{x}{2}}} + \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} =$$

$$= \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1-t^2}{1+t^2} \quad \text{con } t = \tan \frac{x}{2}$$

FORMULE DI PROSTAFERESI E DI WERNER (CENNI)

PROSTAFERESI

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

WERNER

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

DIM.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta \Rightarrow \sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

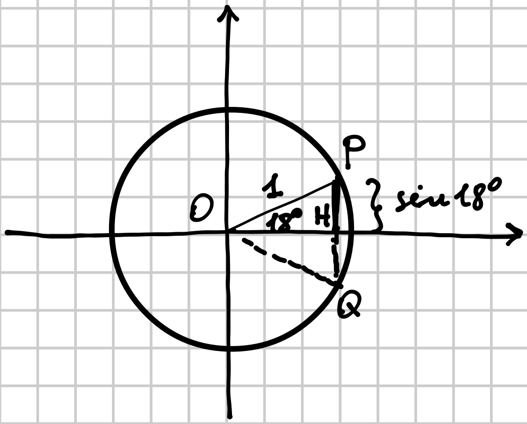
$$\Downarrow$$

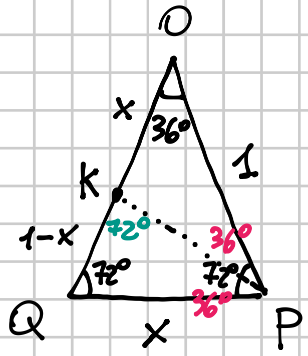
$$\begin{cases} \alpha+\beta = p \\ \alpha-\beta = q \end{cases} \qquad \begin{cases} \alpha = \dfrac{p+q}{2} \\ \beta = \dfrac{p-q}{2} \end{cases} \Rightarrow \sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$2\alpha \quad // \quad = p+q$ | SOMMA

$2\beta = p-q$ | DIFFERENZA

L'ANGOLO DI 18°





$\overline{PK} = \overline{OK} = \overline{QP} = x$

$\overset{\triangle}{QPK}$ è simile a $\overset{\triangle}{QPO}$ (angoli interni congruenti)

$\overline{KQ} : \overline{QP} = \overline{QP} : \overline{PO}$

$(1-x) : x = x : 1$

$x^2 = 1 - x$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1+4}}{2} = \begin{cases} \dfrac{-1-\sqrt{5}}{2} & \text{N.A.} \\ \dfrac{\sqrt{5}-1}{2} & \text{(SEZIONE AUREA)} \end{cases}$

Quindi $\overline{PQ} = \dfrac{\sqrt{5}-1}{2}$

da cui

$$\boxed{\sin 18° = \frac{\overline{PQ}}{2} = \frac{\sqrt{5}-1}{4}}$$