255 $\sqrt{3}\sin 3x + \cos 3x = \sqrt{3}$

$3x = y$

$\sqrt{3}\sin y + \cos y = \sqrt{3}$

$\sin y = \frac{2t}{1+t^2} \quad \cos y = \frac{1-t^2}{1+t^2}$

$t = \tan\frac{y}{2}$

CONTROLLO $(y = \pi + 2k\pi)$

$\sqrt{3}\cdot 0 - 1 = \sqrt{3}$ FALSO!

$$\sqrt{3}\cdot\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \sqrt{3}$$

$$\frac{2\sqrt{3}t + 1 - t^2}{\cancel{1+t^2}} = \frac{\sqrt{3}(1+t^2)}{\cancel{1+t^2}}$$

$$2\sqrt{3}t + 1 - t^2 = \sqrt{3} + \sqrt{3}t^2$$

$$\sqrt{3}t^2 + t^2 - 2\sqrt{3}t + \sqrt{3} - 1 = 0$$

$$(\sqrt{3}+1)t^2 - 2\sqrt{3}t + (\sqrt{3}-1) = 0$$

$$\frac{\Delta}{4} = (-\sqrt{3})^2 - (\sqrt{3}+1)(\sqrt{3}-1) = 3 - 2 = 1$$

$$t = \frac{\sqrt{3}\pm 1}{\sqrt{3}+1} = \begin{cases} \frac{\sqrt{3}-1}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3} \\ \frac{\sqrt{3}+1}{\sqrt{3}+1} = 1 \end{cases}$$

$\tan\frac{y}{2} = 2-\sqrt{3} \quad \vee \quad \tan\frac{y}{2} = 1$

$\frac{y}{2} = \frac{\pi}{12} + k\pi \quad \vee \quad \frac{y}{2} = \frac{\pi}{4} + k\pi$

$y = \frac{\pi}{6} + 2k\pi \quad \vee \quad y = \frac{\pi}{2} + 2k\pi$

$3x = \frac{\pi}{6} + 2k\pi \quad \vee \quad 3x = \frac{\pi}{2} + 2k\pi$

$$\boxed{x = \frac{\pi}{18} + \frac{2}{3}k\pi \quad \vee \quad x = \frac{\pi}{6} + \frac{2}{3}k\pi}$$

EQUAZIONI OMOGENEE DI 2° GRADO

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x = 0 \qquad (a, b, c \neq 0)$$

DIVIDO PER $\cos^2 x$

$$a \underbrace{\frac{\sin^2 x}{\cos^2 x}}_{\tan^2 x} + b \underbrace{\frac{\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x}}_{\tan x} + c \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$a \tan^2 x + b \tan x + c = 0$$

Posso dividere per $\cos^2 x$ perché sicuramente $\cos x \neq 0$. Infatti, se vale

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x = 0$$

e se fosse $\cos x = 0$, sarebbe

$$a \sin^2 x + b \sin x \cdot 0 + c \cdot 0 = 0$$

$$a \sin^2 x = 0$$

$$\Downarrow$$

$\sin x = 0$ MA QUESTO È IMPOSSIBILE PERCHÉ QUANDO $\cos x = 0$ SI HA CHE $\sin x = \pm 1$

284 $\sin^2 x - \sin x \cos x - 2\cos^2 x = 0$

divido per $\cos^2 x$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x} - 2\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$\tan^2 x - \tan x - 2 = 0 \qquad \Delta = 1 + 8 = 9$$

$$\tan x = \frac{1 \pm 3}{2} = \begin{cases} -1 \Rightarrow \\ 2 \Rightarrow \end{cases} \quad \boxed{\begin{array}{l} x = -\frac{\pi}{4} + k\pi \\ \vee \\ x = \arctan 2 + k\pi \end{array}}$$

290 $3\sin^2 x + 2\sin x \cos x = 2 - 3\cos^2 x$

$$3\sin^2 x + 2\sin x \cos x + 3\cos^2 x = 2$$

NON È 0

Per ricondurre l'equazione a una omogenea:

$$3\sin^2 x + 2\sin x \cos x + 3\cos^2 x = 2\overbrace{(\cos^2 x + \sin^2 x)}^{1}$$

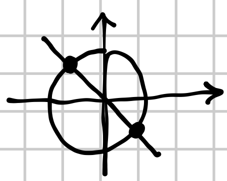
$$3\sin^2 x - 2\sin^2 x + 2\sin x\cos x + 3\cos^2 x - 2\cos^2 x = 0$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{2\sin x \cos x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = 0$$

$$\tan^2 x + 2\tan x + 1 = 0 \Rightarrow (\tan x + 1)^2 = 0$$

$$\tan x = -1 \qquad \boxed{x = -\frac{\pi}{4} + k\pi}$$

Si poteva anche osservare subito che $\sin^2 x + 2\sin x \cos x + \cos^2 x = 0$ si scrive $(\sin x + \cos x)^2 = 0 \Rightarrow \sin x = -\cos x$



$x = -\frac{\pi}{4} + k\pi$

295 $\cos x \sin\left(\frac{2}{3}\pi + x\right) - \sqrt{3}\cos^2 x = \sin x \cos\left(x - \frac{\pi}{6}\right)$

$$\cos x\left[\sin\frac{2}{3}\pi\cos x + \cos\frac{2}{3}\pi\sin x\right] - \sqrt{3}\cos^2 x =$$

$$= \sin x\left[\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right]$$

$$\cos x\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right] - \sqrt{3}\cos^2 x = \sin x\left[\cos x\cdot\frac{\sqrt{3}}{2} + \sin x\cdot\frac{1}{2}\right]$$

$$\frac{\sqrt{3}}{2}\cos^2 x - \frac{1}{2}\sin x\cos x - \sqrt{3}\cos^2 x = \frac{\sqrt{3}}{2}\sin x\cos x + \frac{1}{2}\sin^2 x$$

$$\sqrt{3}\cos^2 x - \sin x\cos x - 2\sqrt{3}\cos^2 x = \sqrt{3}\sin x\cos x + \sin^2 x$$

$$\sin^2 x + (\sqrt{3}+1)\sin x\cos x + \sqrt{3}\cos^2 x = 0$$

$$\tan^2 x + (\sqrt{3}+1)\tan x + \sqrt{3} = 0 \qquad \Delta = (\sqrt{3}+1)^2 - 4\sqrt{3} =$$

$$= 3+1+2\sqrt{3} - 4\sqrt{3} =$$

$$= 3+1-2\sqrt{3} = (\sqrt{3}-1)^2$$

$$\tan x = \frac{-\sqrt{3}-1 \pm (\sqrt{3}-1)}{2} = \begin{cases} \dfrac{-\sqrt{3}\cancel{-1}-\sqrt{3}\cancel{+1}}{2} = -\sqrt{3} \\[2ex] \dfrac{\cancel{-\sqrt{3}}-1\cancel{+\sqrt{3}}-1}{2} = -1 \end{cases}$$

$$\tan x = -\sqrt{3} \quad \vee \quad \tan x = -1$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{x = -\frac{\pi}{3} + k\pi \quad \vee \quad x = -\frac{\pi}{4} + k\pi}$$