1/12/2022

371 $\sin\left(x+\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}\right)=1$

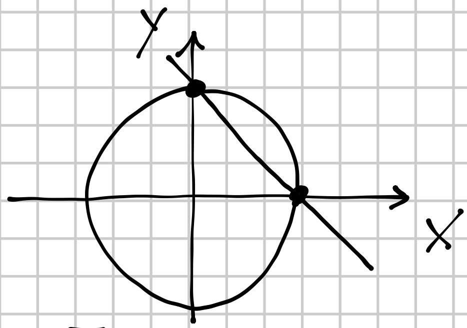
$x+\frac{\pi}{3}=t$

$\sin t+\cos t=1$

$\begin{cases} Y+X=1 \\ X^2+Y^2=1 \end{cases}$ $\begin{cases} Y=1-X \\ X^2+(1-X)^2-1=0 \end{cases}$ $\begin{cases} Y=1-X \\ X^2+\cancel{1}+X^2-2X-\cancel{1}=0 \end{cases}$

$\begin{cases} // \\ 2X^2-2X=0 \end{cases}$ $\begin{cases} \\ 2X(X-1)=0 \end{cases}$ $\to X=0$, $\to X=1$

$\begin{cases} X=0 \\ Y=1 \end{cases} \vee \begin{cases} X=1 \\ Y=0 \end{cases}$



$t=0+2k\pi=2k\pi$

$\vee$

$t=\frac{\pi}{2}+2k\pi$

$t=x+\frac{\pi}{3}$

$x+\frac{\pi}{3}=2k\pi \quad \vee \quad x+\frac{\pi}{3}=\frac{\pi}{2}+2k\pi$

$$\boxed{x=-\frac{\pi}{3}+2k\pi \quad \vee \quad x=\frac{\pi}{6}+2k\pi}$$

$\frac{\pi}{2}-\frac{\pi}{3}$

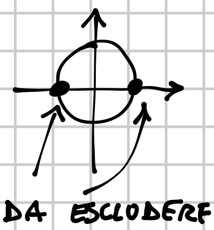
381 $\dfrac{\sin^2 \frac{x}{2}}{\sin^2 x} + \cos x = 1$ $\left[\dfrac{\pi}{4} + k\dfrac{\pi}{2}\right]$

C.E.

$\sin^2 x \neq 0$

$\sin x \neq 0$

$x \neq k\pi$



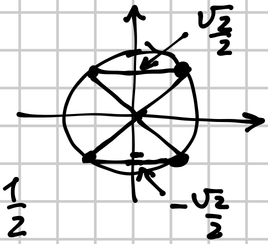
$$\frac{\frac{1-\cos x}{2} + \cos x \cdot \sin^2 x}{\cancel{\sin^2 x}} = \frac{\sin^2 x}{\cancel{\sin^2 x}}$$

$$1 - \cos x + 2\cos x \sin^2 x = 2\sin^2 x$$

$$1 - \cos x + 2\cos x \sin^2 x - 2\sin^2 x = 0$$

$$1 - \cos x - 2\sin^2 x(-\cos x + 1) = 0$$

$$(1-\cos x)(1 - 2\sin^2 x) = 0$$



$1 - \cos x = 0 \quad \vee \quad 1 - 2\sin^2 x = 0$

$\sin^2 x = \frac{1}{2}$

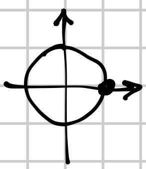
$\sin x = \pm \frac{\sqrt{2}}{2}$

$\downarrow$

$x = \frac{\pi}{4} + k\frac{\pi}{2}$

$\cos x = 1 \quad \vee \quad \cos 2x = 0$

$x = 2k\pi$ N.A. per C.E.



$2x = \frac{\pi}{2} + k\pi$

$$\boxed{x = \frac{\pi}{4} + k\frac{\pi}{2}}$$

Confrontando con le C.E. $(x \neq k\pi)$ devo escludere le soluzioni del tipo $x = 2k\pi$

L'unica soluzione è

$$\frac{\sqrt{3}\cos x + 2\sin x}{\cot x} = \frac{1+\sin^2 x - \cos 2x}{\cos x}$$

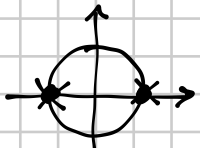
421

C.E.

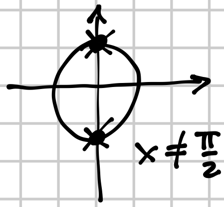
$\cot x$ deve esistere! $\Rightarrow x \neq k\pi$ $\left(\cot x = \frac{\cos x}{\sin x} \text{ e } \sin x \neq 0\right)$

$\cot x$ deve essere $\neq 0 \Rightarrow \cot x \neq 0$ $\quad x \neq \frac{\pi}{2} + k\pi$

$\Rightarrow x \neq k\frac{\pi}{2}$

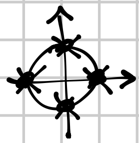


$x \neq k\pi$



$x \neq \frac{\pi}{2} + k\pi$

INSIEME $\Rightarrow$ $x \neq k\frac{\pi}{2}$



(anche con $\tan x$ al denominatore sarebbe $x \neq k\frac{\pi}{2}$)

$\cos x \neq 0$ già rientra nel precedente!

$$\frac{\sqrt{3}\cos x + 2\sin x}{\frac{\cancel{\cos x}}{\sin x}} = \frac{1+\sin^2 x - \cos 2x}{\cancel{\cos x}}$$

$$\sqrt{3}\sin x\cos x + 2\sin^2 x = 1 + \sin^2 x - (1 - 2\sin^2 x)$$

$$\sqrt{3}\sin x\cos x + \cancel{2\sin^2 x} = \cancel{1} + \sin^2 x - \cancel{1} + \cancel{2\sin^2 x}$$

$$\sin^2 x - \sqrt{3}\sin x\cos x = 0$$

$$\sin x\left(\sin x - \sqrt{3}\cos x\right) = 0$$

$\sin x = 0$
IMPOSS. PER C.E.

$\sin x - \sqrt{3}\cos x = 0$

$\sin x = \sqrt{3}\cos x$ divido per $\cos x$ perché $\cos x \neq 0$ per C.E.

$\frac{\sin x}{\cos x} = \sqrt{3} \Rightarrow \tan x = \sqrt{3}$

$$\boxed{x = \frac{\pi}{3} + k\pi}$$

437 $\log_3(3\sin x) - \log_3\cos x = \log_3\sqrt{27}$ $\left[\frac{\pi}{3} + 2k\pi\right]$

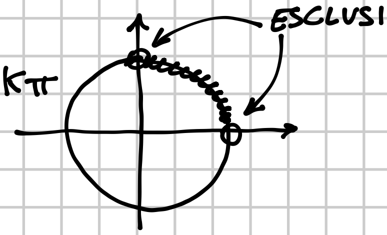
$\log_a x - \log_a y = \log_a \frac{x}{y}$

applico le propr. dei logaritmi

gli argomenti dei logaritmi devono essere > 0

C.E. $\begin{cases} 3\sin x > 0 \Rightarrow \sin x > 0 \\ \cos x > 0 \end{cases}$

$2k\pi < x < \frac{\pi}{2} + 2k\pi$



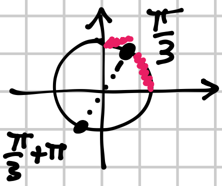
$\log_3 \frac{3\sin x}{\cos x} = \log_3 (3\sqrt{3})$

passo agli argomenti

$\frac{\cancel{3}\sin x}{\cos x} = \cancel{3}\sqrt{3}$

$\tan x = \sqrt{3}$

$\Downarrow$



INTERSECO CON LE C.E.

$\begin{cases} x = \frac{\pi}{3} + k\pi \\ 2k\pi < x < \frac{\pi}{2} + 2k\pi \end{cases}$ C.E.

$\Downarrow$

$$\boxed{x = \frac{\pi}{3} + 2k\pi}$$