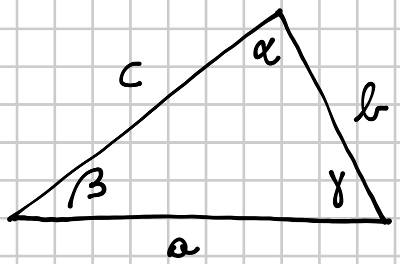
TEOREMA DEL COSENO (DI CARNOT)

(È una generalizzazione del teorema di Pitagora)

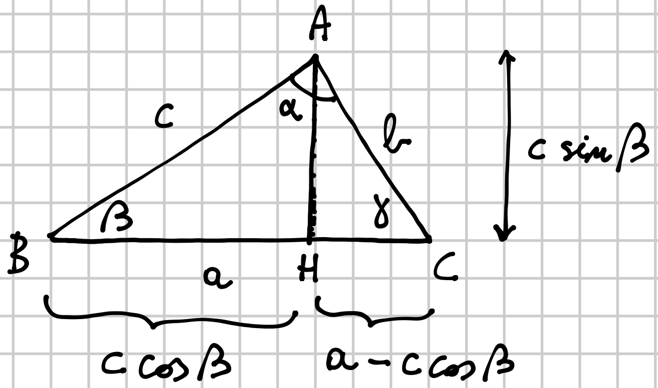


$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

DIMOSTRAZIONE



$$\overline{AC}^2 = \overline{AH}^2 + \overline{HC}^2 \quad \text{(teorema di Pitagora)}$$

$$b^2 = (c\sin\beta)^2 + (a - c\cos\beta)^2$$

$$b^2 = c^2 \cdot \sin^2\beta + a^2 + c^2\cos^2\beta - 2ac\cos\beta$$

$$b^2 = a^2 + c^2\underbrace{(\sin^2\beta + \cos^2\beta)}_{1} - 2ac\cos\beta$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

219 $a = 12, \quad b = 6, \quad \gamma = \frac{\pi}{3}. \qquad c? \qquad [6\sqrt{3}]$

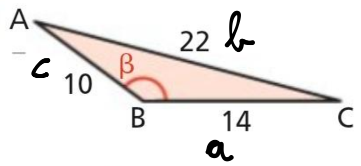
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = 144 + 36 - 144 \cdot \underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}} = 144 + 36 - 72 = 108$$

$$c = \sqrt{108} = \sqrt{2^2 \cdot 3^3} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$$

108	2
54	$3^3 \cdot 2$
1	

228



β?

[≃ 132°]

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$484 = 196 + 100 - 280\cos\beta$$

$$280\cos\beta = -188$$

$$\cos\beta = -\frac{188}{280}$$

$$\beta = \arccos\left(-\frac{188}{280}\right) =$$

$$= 132{,}177\ldots^\circ \simeq 132^\circ$$