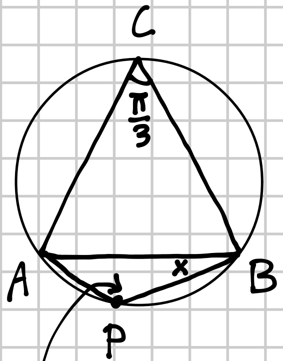
208 Considera il triangolo equilatero ABC e la circonferenza a esso circoscritta di raggio r. Sull'arco $\widehat{AB}$ che non contiene C prendi il punto P. Calcola $A\hat{B}P$ in modo che l'area del quadrilatero $APBC$ sia i $\frac{4}{3}$ dell'area del triangolo equilatero. $\left[\frac{\pi}{6}\right]$



$\pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$\mathcal{A}_{APBC} = \frac{4}{3}\mathcal{A}_{ABC}$ || equazione da imporre alla fine

TH. DELLA CORDA $\Rightarrow$ $\overline{AB} = 2r \cdot \sin 60° = 2r \cdot \frac{\sqrt{3}}{2} = r\sqrt{3}$

$P\hat{A}B = \pi - \frac{2}{3}\pi - x = \frac{\pi}{3} - x \Rightarrow 0 < x < \frac{\pi}{3}$

$\mathcal{A}_{APB} = \frac{1}{2}\underbrace{\overline{AB}}_{\text{lo so}} \cdot \underbrace{\overline{PB}}_{\text{DA TROVARE!}} \cdot \sin x =$

$\overline{PB} = 2r \cdot \sin P\hat{A}B = 2r \cdot \sin\left(\frac{\pi}{3} - x\right)$

$\frac{1}{\cancel{2}} r\sqrt{3} \cdot \cancel{2} r \sin\left(\frac{\pi}{3} - x\right) \cdot \sin x =$

$= r^2\sqrt{3}\sin\left(\frac{\pi}{3} - x\right) \cdot \sin x$

$\mathcal{A}_{ABC} = \frac{1}{2}\overline{AC} \cdot \overline{CB} \cdot \sin\frac{\pi}{3} = \frac{1}{2}(r\sqrt{3})^2 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}r^2$

$\mathcal{A}_{APBC} = \mathcal{A}_{APB} + \mathcal{A}_{ABC}$

USO L'EQUAZIONE DEL PROBLEMA

$\mathcal{A}_{APB} + \mathcal{A}_{ABC} = \frac{4}{3}\mathcal{A}_{ABC}$

$\mathcal{A}_{APB} = \frac{4}{3}\mathcal{A}_{ABC} - \mathcal{A}_{ABC}$

$\mathcal{A}_{APB} = \frac{1}{3}\mathcal{A}_{ABC}$

$\cancel{r^2}\cancel{\sqrt{3}}\sin\left(\frac{\pi}{3} - x\right) \cdot \sin x = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3\sqrt{3}}}{4}\cancel{r^2}$

$4\sin\left(\frac{\pi}{3} - x\right) \cdot \sin x = 1 \qquad 0 < x < \frac{\pi}{3}$

$$4\sin\left(\frac{\pi}{3}-x\right)\cdot\sin x = 1 \qquad 0 < x < \frac{\pi}{3}$$

$$4\left[\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right]\cdot\sin x = 1$$

$$4\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right]\cdot\sin x = 1$$

$$2\sqrt{3}\sin x\cos x - 2\sin^2 x = 1$$

$$2\sqrt{3}\sin x\cos x - 2\sin^2 x = \sin^2 x + \cos^2 x$$

$$3\sin^2 x - 2\sqrt{3}\sin x\cos x + \cos^2 x = 0$$

$\downarrow$ divido per $\cos^2 x$

$$3\tan^2 x - 2\sqrt{3}\tan x + 1 = 0$$

$$\frac{\Delta}{4} = 3-3 = 0 \qquad \left(\sqrt{3}\tan x - 1\right)^2 = 0 \quad \Rightarrow \quad \tan x = \frac{1}{\sqrt{3}}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

$$\begin{cases} x = \frac{\pi}{6} + k\pi \\ 0 < x < \frac{\pi}{3} \end{cases}$$

$$\Downarrow$$

$$\boxed{x = \frac{\pi}{6}}$$