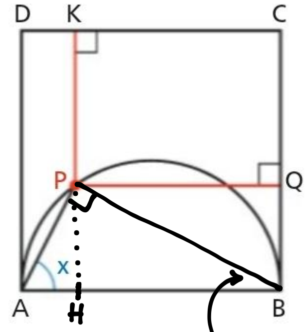
**341** Il quadrato $ABCD$ nella figura ha il lato di lunghezza 2 e il punto $P$ appartiene alla semicirconferenza di diametro $AB$.

**a.** Risolvi l'equazione $\frac{\overline{PK}}{\overline{PQ}} = \frac{3}{4}$.

**b.** Esprimi la funzione $f(x) = \overline{PK} + \overline{PQ}$ al variare di $P$ sulla semicirconferenza e rappresentala in un periodo evidenziando la parte relativa al problema.

$\left[\text{a)} x = \arctan 2; \text{ b) } y = 3 - \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)\right]$




$\frac{\pi}{2} - x$

$0 \leq x \leq \frac{\pi}{2}$

$\overline{PQ} = \overline{AB} - \overline{AH} = 2 - \overline{AP}\cos x$

$\overline{AP} = \overline{AB}\cos x$

$= 2 - 2\cos x \cdot \cos x = 2 - 2\cos^2 x$

$\overline{PK} = \overline{KH} - \overline{PH} = 2 - \overline{AP}\cdot \sin x =$

$= 2 - 2\cos x \cdot \sin x$

$\frac{\overline{PK}}{\overline{PQ}} = \frac{3}{4}$ $\qquad \frac{2 - 2\cos x \sin x}{2 - 2\cos^2 x} = \frac{3}{4}$ $\qquad 0 \leq x \leq \frac{\pi}{2}$

DEVO ESCLUDERE $x = 0$

$\frac{\cancel{2}(1 - \cos x \sin x)}{\cancel{2}(1 - \cos^2 x)} = \frac{3}{4}$ $\qquad 0 < x \leq \frac{\pi}{2}$

$4(1 - \cos x \sin x) = 3(1 - \cos^2 x)$

$4 - 4\cos x \sin x = 3 - 3\cos^2 x$

$3\cos^2 x - 4\cos x \sin x + 1 = 0$

$3\cos^2 x - 4\cos x \sin x + \sin^2 x + \cos^2 x = 0$

$\sin^2 x - 4\cos x \sin x + 4\cos^2 x = 0$ $\quad$ divido per $\cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{4\cos x \sin x}{\cos^2 x} + \frac{4\cos^2 x}{\cos^2 x} = 0$

$\tan^2 x - 4\tan x + 4 = 0$ $\qquad (\tan x - 2)^2 = 0 \Rightarrow \tan x = 2$

$\boxed{x = \arctan 2}$

$0 < x \leq \frac{\pi}{2}$

$$f(x) = \overline{PK} + \overline{PQ} = 2 - 2\cos x \sin x + 2 - 2\cos^2 x =$$

$$= 4 - 2\cos x \sin x - 2\cos^2 x =$$

$$= 4 - \sin 2x - \left(2\cos^2 x - 1 + 1\right) =$$

$$= 4 - \sin 2x - \cos 2x - 1 = 3 - \sin 2x - \cos 2x =$$

$$= 3 - \left(\sin 2x + \cos 2x\right)$$

$$\sin 2x + \cos 2x = r \sin\left(2x + \varphi\right) \qquad r > 0$$

$$= r\left[\sin 2x \cdot \cos\varphi + \cos 2x \cdot \sin\varphi\right] =$$

$$= \underbrace{r \cdot \cos\varphi}_{1} \cdot \sin 2x + \underbrace{r \cdot \sin\varphi}_{1} \cdot \cos 2x$$

$$\begin{cases} r \cdot \cos\varphi = 1 \\ r \cdot \sin\varphi = 1 \end{cases} \qquad r^2\cos^2\varphi + r^2\sin^2\varphi = 1 + 1 \quad \text{(SOMMA DEI QUADRATI)}$$

$$r^2\underbrace{\left(\cos^2\varphi + \sin^2\varphi\right)}_{1} = 2$$

$$\Downarrow$$

$$r = \sqrt{2}$$

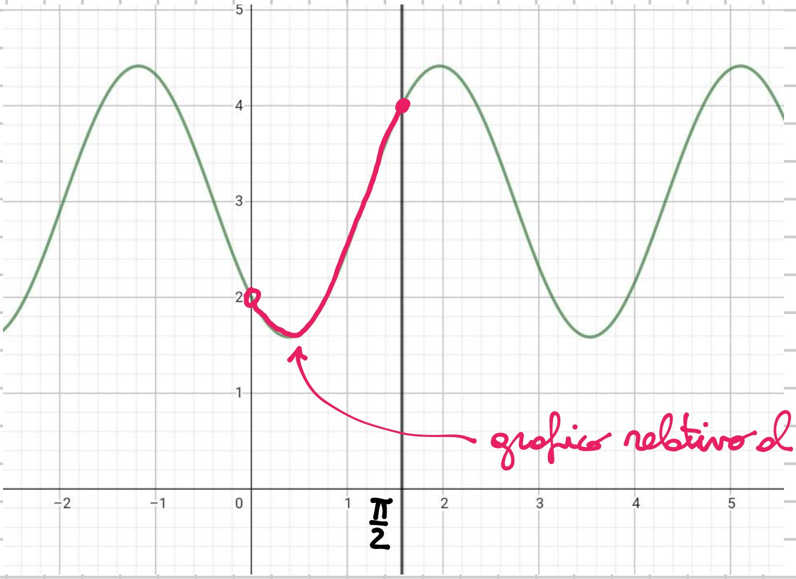
$$\begin{cases} \sqrt{2}\cos\varphi = 1 \\ \sqrt{2}\sin\varphi = 1 \end{cases} \qquad \begin{cases} \cos\varphi = \dfrac{1}{\sqrt{2}} \\ \sin\varphi = \dfrac{1}{\sqrt{2}} \end{cases} \qquad \begin{cases} \cos\varphi = \dfrac{\sqrt{2}}{2} \\ \sin\varphi = \dfrac{\sqrt{2}}{2} \end{cases} \Rightarrow \varphi = \frac{\pi}{4}$$

in definitiva $\quad \sin 2x + \cos 2x = \sqrt{2}\sin\left(2x + \dfrac{\pi}{4}\right)$

$$f(x) = 3 - \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$$

PASSAGGI $\quad \sin x \qquad \sin\left(x + \dfrac{\pi}{4}\right) \qquad \sin\left(2x + \dfrac{\pi}{4}\right) \qquad \sqrt{2}\sin\left(2x + \dfrac{\pi}{4}\right)$

$$-\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right) \qquad -\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right) + 3$$



$$y = 3 - \sqrt{2}\,\sin\left(2x + \frac{\pi}{4}\right)$$

grafico relativo al problema $0 < x \le \frac{\pi}{2}$