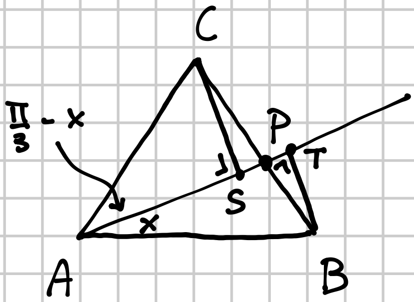
333 Sono dati il triangolo equilatero ABC di lato 1 e la semiretta di origine A che incontra il lato BC nel punto P. Su tale semiretta, considera il punto S proiezione di C e il punto T proiezione di B. Indicato con x l'angolo $B\widehat{A}P$, determina la funzione:

$$f(x) = \overline{AB}^2 - \overline{CS}^2 - \overline{BT}^2.$$

Trova x in modo che $f(x) = \frac{\sqrt{3}}{4}$.

$$\left[f(x) = \frac{\cos 2x + \sqrt{3}\sin 2x}{4};\ \frac{\pi}{12} \vee \frac{\pi}{4}\right]$$



$\overline{AB} = 1$

$0 \le x \le \frac{\pi}{3}$

$\overline{CS} = \overline{AC} \cdot \sin\left(\frac{\pi}{3} - x\right) = \sin\left(\frac{\pi}{3} - x\right)$

$\overline{BT} = \overline{AB} \cdot \sin x = \sin x$

$$f(x) = 1^2 - \sin^2\left(\frac{\pi}{3} - x\right) - \sin^2 x =$$

$$= 1 - \left[\sin\frac{\pi}{3} \cdot \cos x - \cos\frac{\pi}{3}\sin x\right]^2 - \sin^2 x =$$

$$= 1 - \left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right]^2 - \sin^2 x =$$

$$= 1 - \left(\frac{3}{4}\cos^2 x + \frac{1}{4}\sin^2 x - \frac{\sqrt{3}}{2}\cos x \sin x\right) - \sin^2 x =$$

$$= 1 - \frac{3}{4}\cos^2 x - \frac{1}{4}\sin^2 x + \frac{\sqrt{3}}{2}\cos x \sin x - \sin^2 x =$$

$$= 1 - \frac{3}{4}\cos^2 x - \frac{5}{4}\sin^2 x + \frac{\sqrt{3}}{2}\cos x \sin x =$$

$$= \frac{4 - 3\cos^2 x - 5(1 - \cos^2 x) + 2\sqrt{3}\cos x \sin x}{4} =$$

$$= \frac{4 - 3\cos^2 x - 5 + 5\cos^2 x + \sqrt{3}\sin 2x}{4} =$$

$$= \frac{2\cos^2 x - 1 + \sqrt{3}\sin 2x}{4} = \frac{\cos 2x + \sqrt{3}\sin 2x}{4}$$

$$\frac{\cos 2x + \sqrt{3}\sin 2x}{\cancel{4}} = \frac{\sqrt{3}}{\cancel{4}} \qquad 0 \le x \le \frac{\pi}{3}$$

$$f(x) = \frac{\sqrt{3}}{4}$$

$$\cos 2x + \sqrt{3}\sin 2x = \sqrt{3} \qquad X = \cos 2x \quad Y = \sin 2x$$

$$\begin{cases} X + \sqrt{3}Y = \sqrt{3} \\ X^2 + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = \sqrt{3} - \sqrt{3}Y \\ (\sqrt{3} - \sqrt{3}Y)^2 + Y^2 = 1 \end{cases}$$

$$3 + 3Y^2 - 6Y + Y^2 - 1 = 0$$

$$4Y^2 - 6Y + 2 = 0$$

$$2Y^2 - 3Y + 1 = 0 \qquad \Delta = 9 - 8 = 1$$

$$Y = \frac{3 \pm 1}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases} \lor \begin{cases} X = \frac{\sqrt{3}}{2} \\ Y = \frac{1}{2} \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

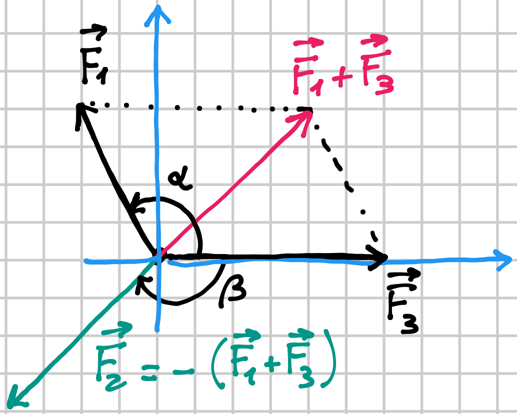
$$2x = \frac{\pi}{2} \qquad 2x = \frac{\pi}{6}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} x = \frac{\pi}{4} \lor x = \frac{\pi}{12} \\ 0 \le x \le \frac{\pi}{3} \end{cases} \Rightarrow \boxed{x = \frac{\pi}{4} \lor x = \frac{\pi}{12}}$$

375 **COMPOSIZIONE DI FORZE** Tre forze, $\vec{F}_1$, $\vec{F}_2$ e $\vec{F}_3$, i cui moduli sono 8 N, 4 N e 10 N, sono applicate su una stessa massa puntiforme. Determina gli angoli che $\vec{F}_1$ e $\vec{F}_2$ formano con $\vec{F}_3$, sapendo che le tre forze si equilibrano.

[158°; 130°]



Nel sistema di rif. fissato ho

$\vec{F}_3=(10,0)$ (omettiamo l'u.d.m. N)

$\vec{F}_1=(8\cos\alpha,\ 8\sin\alpha)$

$\vec{F}_2=(4\cos\beta,\ -4\sin\beta)$

$$\vec{F}_1+\vec{F}_2+\vec{F}_3=\vec{0}$$

$$\Rightarrow \begin{cases} 10+8\cos\alpha+4\cos\beta=0 \\ 8\sin\alpha-4\sin\beta=0 \end{cases} \qquad \begin{cases} 5+4\cos\alpha+2\cos\beta=0 \\ \cancel{4}\sin\beta=\overset{2}{\cancel{8}}\sin\alpha \end{cases}$$

$$\begin{cases} 5+4\cos\alpha+2\cos\beta=0 \\ \sin\beta=2\sin\alpha \end{cases} \qquad \begin{cases} 2\cos\beta=-5-4\cos\alpha \\ \sin\beta=2\sin\alpha \end{cases}$$

DA QUI SI VEDE CHE $\cos\beta<0$ $(-4\leq 4\cos\alpha\leq 4)$

elevo al quadrato

$$\begin{cases} 4\cos^2\beta=25+16\cos^2\alpha+40\cos\alpha \\ \sin^2\beta=4\sin^2\alpha \Rightarrow 1-\cos^2\beta=4\sin^2\alpha \Rightarrow \cos^2\beta=1-4\sin^2\alpha \end{cases}$$

$$4(1-4\sin^2\alpha)=25+16\cos^2\alpha+40\cos\alpha$$

$$4-16\sin^2\alpha=25+16\cos^2\alpha+40\cos\alpha$$

$$16\cos^2\alpha+16\sin^2\alpha+40\cos\alpha+25-4=0$$

$$16\underbrace{(\cos^2\alpha+\sin^2\alpha)}_{1}+40\cos\alpha+21=0$$

$$16+40\cos\alpha+21=0 \qquad \cos\alpha=-\frac{37}{40}$$

$$\alpha=\arccos\left(-\frac{37}{40}\right)=157{,}6\ldots°$$

$$\simeq 158°$$

$\sin\beta = 2\sin\alpha = 2\sin(157{,}6°\ldots)$

$$\Downarrow$$

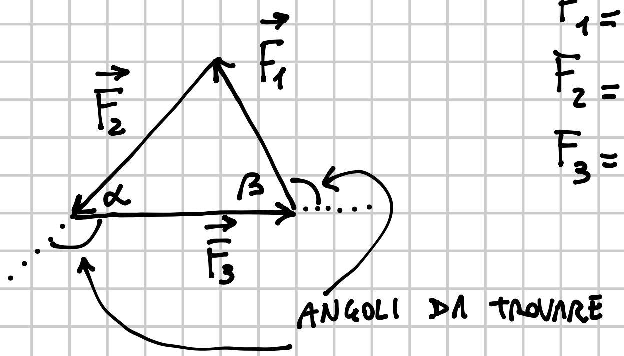
$\beta = \arcsin(2\sin(157{,}6°\ldots)) = 49{,}459\ldots°$ N.Acc. perché $\cos\beta < 0$

$\vee$

$\beta = 180° - 49{,}459\ldots° = 130{,}54\ldots° \simeq \boxed{131°}$

RISOLUZIONE ALTERNATIVA

Dato che $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$ si ha



$F_1 = 8\,N$

$F_2 = 4\,N$

$F_3 = 10\,N$

Applico il teorema del coseno:

1) $F_2^2 = F_1^2 + F_3^2 - 2F_1F_3\cos\beta$

$16 = 64 + 100 - 2\cdot 8\cdot 10\cdot\cos\beta \qquad 16 = 164 - 160\cos\beta$

$160\cos\beta = 148 \qquad 40\cos\beta = 37 \qquad \cos\beta = \frac{37}{48}$

ANGOLO $\widehat{F_1F_3} = 180° - \arccos\frac{37}{48} = 157{,}6683\ldots° \simeq \boxed{158°}$

2) $F_1^2 = F_2^2 + F_3^2 - 2F_2F_3\cos\alpha$

$64 = 16 + 100 - 2\cdot 4\cdot 10\cdot\cos\alpha \qquad 64 = 116 - 80\cos\alpha \qquad 16 = 29 - 20\cos\alpha$

$20\cos\alpha = 13 \qquad \cos\alpha = \frac{13}{20}$

ANGOLO $\widehat{F_2F_3} = 180° - \arccos\frac{13}{20} = 130{,}5416\ldots° \simeq \boxed{131°}$