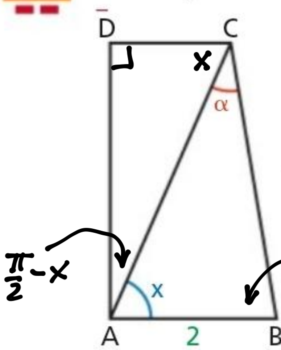
215 Nel trapezio rettangolo in figura, determina x in modo che si abbia $5\overline{AD} + \overline{DC} > 2$. $\left[\frac{\pi}{2} - \alpha < x < \frac{\pi}{2}\right]$



$\cos\alpha = \frac{3}{\sqrt{13}} \Rightarrow \alpha = \arccos\frac{3}{\sqrt{13}}$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{13}} = \frac{2}{\sqrt{13}}$$

$$\begin{cases} 0 < x < \frac{\pi}{2} \\ 0 < x < \pi - \alpha \end{cases} \Rightarrow 0 < x < \frac{\pi}{2} \quad \text{perché } \pi - \arccos\frac{3}{\sqrt{13}} > \frac{\pi}{2}$$

TH. SENI $\Rightarrow \dfrac{\overline{AC}}{\sin\hat{B}} = \dfrac{\overline{AB}}{\sin\alpha} \Rightarrow \overline{AC} = \dfrac{2}{\sin\alpha}\cdot\sin(\pi - x - \alpha) =$

$$= \frac{2\sin(x+\alpha)}{\sin\alpha} = \frac{2\left[\sin x\cos\alpha + \cos x\sin\alpha\right]}{\frac{2}{\sqrt{13}}} =$$

$$= \frac{\cancel{2}\left[\sin x\cdot\frac{3}{\sqrt{13}} + \cos x\,\frac{2}{\sqrt{13}}\right]}{\frac{\cancel{2}}{\sqrt{13}}} = \left[\sin x\cdot\frac{3}{\cancel{\sqrt{13}}} + \cos x\cdot\frac{2}{\cancel{\sqrt{13}}}\right]\cdot\cancel{\sqrt{13}} =$$

$$= 3\sin x + 2\cos x$$

$\overline{AD} = \overline{AC}\cdot\cos\left(\frac{\pi}{2}-x\right) = \overline{AC}\cdot\sin x \qquad \overline{DC} = \overline{AC}\cdot\sin\left(\frac{\pi}{2}-x\right) = \overline{AC}\cdot\cos x$

$0 < x < \pi - \alpha$

$5\overline{AD} + \overline{DC} > 2 \qquad 5\overline{AC}\sin x + \overline{AC}\cos x > 2$

$$5(3\sin x + 2\cos x)\sin x + (3\sin x + 2\cos x)\cos x > 2$$

$$15\sin^2 x + 10\cos x\cdot\sin x + 3\sin x\cos x + \cancel{2\cos^2 x} - 2\sin^2 x - \cancel{2\cos^2 x} > 0$$

$\cancel{13}\sin^2 x + \cancel{13}\sin x\cos x > 0 \qquad \sin^2 x + \sin x\cos x > 0$

$\sin^2 x + \sin x \cos x > 0$ ← la risolviamo in $[0, 2\pi]$, poi intersechiamo con $0 < x < \pi - \alpha$

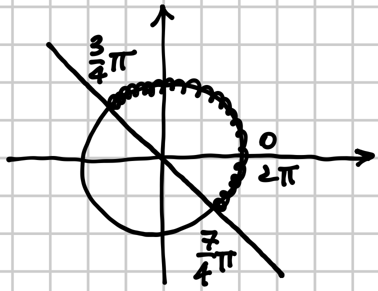
$\sin x(\sin x + \cos x) > 0$

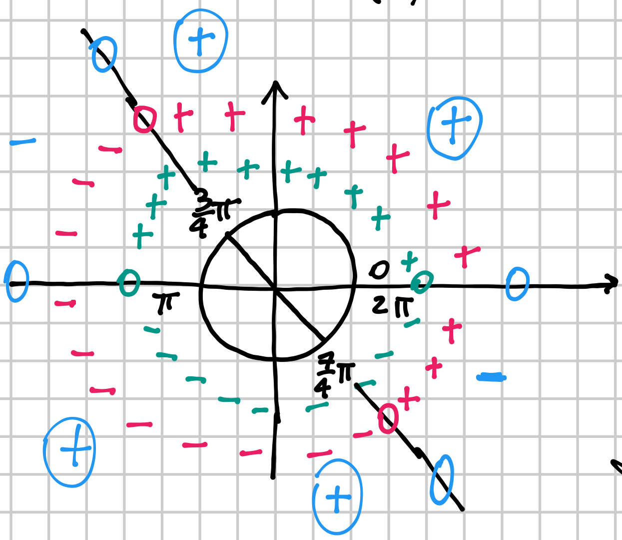
(1) $\sin x > 0$ $\quad \vee \quad$ (2) $\sin x + \cos x > 0$

$0 < x < \pi$

$$\Downarrow$$

$$\begin{cases} X^2 + Y^2 = 1 \\ Y + X > 0 \end{cases} \qquad \begin{cases} X^2 + Y^2 = 1 \\ Y > -X \end{cases}$$





$0 < x < \frac{3}{4}\pi \quad \vee \quad \pi < x < \frac{7}{4}\pi$

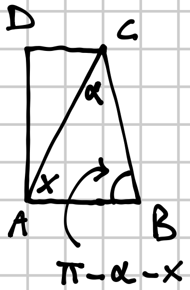
DA INTERSECARE CON $0 < x < \frac{\pi}{2}$

$$\Downarrow$$

$$\boxed{0 < x < \frac{\pi}{2}}$$

OSSERVAZIONE SUL DIVERSO RISULTATO DEL LIBRO

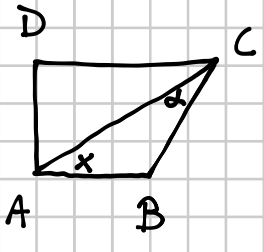
Il libro presuppone implicitamente che l'angolo in $\hat{B}$ debba essere minore di $\frac{\pi}{2}$, per cui



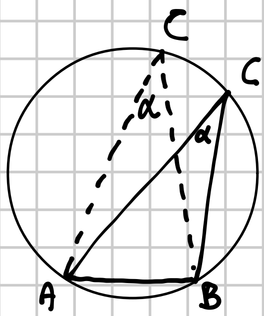
$$\pi - \alpha - x < \frac{\pi}{2} \Rightarrow -x < -\frac{\pi}{2} + \alpha$$

$$\Rightarrow x > \frac{\pi}{2} - \alpha$$

Nella nostra risoluzione, invece, ammettiamo anche trapezi della forma:



$\overline{AB} = 2$ (fissato)



il vertice $\hat{C}$ del triangolo ABC sta sempre sulla circonferenza (α è fissato)