96 $(-2i^2)^3(-i)^8 + \dfrac{8i - 6i^5}{2i} - (-i)^7 i^{15}$ [8]

$$= (-2)^3 \cdot i^6 \cdot i^8 + \frac{\overset{4}{\cancel{8i}}}{\cancel{2i}} - \frac{6i^5}{2i} - (-1)^7 \cdot i^7 \cdot i^{15} =$$

$$= -8i^{14} + 4 - 3i^4 + i^{22} =$$

$$= -8i^2 + 4 - 3 + i^2 = 8 \cancel{+4} \cancel{-3} \cancel{-1} = \boxed{8}$$

FORMULA IMPORTANTE

$$z\bar{z} = (a+ib)(a-ib) = a^2 - (ib)^2 = a^2 - (-b^2) = a^2 + b^2 = |z|^2$$

$z = a + ib$

$$\Downarrow$$

$$z\bar{z} = |z|^2$$

DIVISIONE DI NUMERI COMPLESSI

$$\frac{5+3i}{2-7i} = \frac{5+3i}{2-7i} \cdot \frac{2+7i}{2+7i} = \frac{10 + 35i + 6i + 21i^2}{2^2 + (-7)^2} =$$

MOLTIPLICO "SOPRA E SOTTO" PER IL CONIUGATO DEL DENOMINATORE

$$= \frac{10 + 41i + 21 \cdot (-1)}{4 + 49} = \frac{-11 + 41i}{53} = -\frac{11}{53} + \frac{41}{53} i$$

164 $\frac{3+i}{2-i} - \frac{i-2}{3-i} + (i-1)(i+2) - i = \left[\frac{9i-13}{10}\right]$

$$= \frac{3+i}{2-i} \cdot \frac{2+i}{2+i} - \frac{i-2}{3-i} \cdot \frac{3+i}{3+i} + i^2 + \cancel{2i} - \cancel{i} - 2 - \cancel{i} =$$

$$= \frac{6+3i+2i-1}{4+1} - \frac{3i+i^2-6-2i}{9+1} - 1 - 2 =$$

$$= \frac{5+5i}{5} - \frac{-7+i}{10} - 3 = 1 + i - \left(-\frac{7}{10}\right) - \frac{1}{10}i - 3 =$$

$$= 1 + i + \frac{7}{10} - \frac{1}{10}i - 3 = \left(1 - 3 + \frac{7}{10}\right) + \left(1 - \frac{1}{10}\right)i =$$

$$= \boxed{-\frac{13}{10} + \frac{9}{10}i}$$

165 $(2+3i)(2-3i) - (3+i)^2 + i(\overline{3+2i}) - 6(i+2) =$

$[-5-9i]$

$$= 4 + 9 - (9 + \overset{-1}{i^2} + 6i) + i(3-2i) - 6i - 12 =$$

$$= 13 - 8 - 6i + 3i + 2 - 6i - 12 = \boxed{-5-9i}$$

184 $\dfrac{i(1+i)}{2-i} - \dfrac{3+2i}{1-2i} - (2-i)(\overline{2-3i})\,i = \left[\dfrac{18-42i}{5}\right]$

$$= \frac{i(1+i)}{2-i}\cdot\frac{2+i}{2+i} - \frac{3+2i}{1-2i}\cdot\frac{1+2i}{1+2i} - (2-i)(2+3i)\,i =$$

$$= \frac{(i-1)(2+i)}{4+1} - \frac{3+6i+2i-4}{1+4} - (4+6i-2i+3)\,i =$$

$$= \frac{2i-1-2-i}{5} - \frac{-1+8i}{5} - 7i+4 =$$

$$= \frac{i-3}{5} - \frac{-1+8i}{5} - 7i+4 = \frac{i-3+1-8i-35i+20}{5} =$$

$$= \boxed{\frac{18-42i}{5}}$$

194 $z^2 + |z|^2 = 4 + i$ $\left[\sqrt{2} + \frac{\sqrt{2}}{4}i; -\sqrt{2} - \frac{\sqrt{2}}{4}i\right]$

$z = a + ib \qquad |z| = \sqrt{a^2 + b^2} \qquad a, b \in \mathbb{R}$

$$(a+ib)^2 + |a+ib|^2 = 4 + i$$

$$a^2 + (ib)^2 + 2abi + a^2 + b^2 = 4 + i$$

$$a^2 - \cancel{b^2} + 2abi + a^2 + \cancel{b^2} - 4 - i = 0$$

$$2a^2 - 4 + i(2ab - 1) = 0$$

$$\boxed{z = a + ib = 0 \iff \begin{cases} a = 0 \\ b = 0 \end{cases}}$$

$$\begin{cases} 2a^2 - 4 = 0 \\ 2ab - 1 = 0 \end{cases}$$

$$\begin{cases} a^2 = 2 \\ 2ab - 1 = 0 \end{cases} \qquad \begin{cases} a = \pm\sqrt{2} \\ 2ab = 1 \end{cases} \implies \begin{cases} a = -\sqrt{2} \\ b = \frac{1}{2a} = -\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4} \end{cases}$$

$\vee$

$$\begin{cases} a = \sqrt{2} \\ b = \frac{1}{2a} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4} \end{cases}$$

$$\boxed{z = -\sqrt{2} - \frac{\sqrt{2}}{4}i \quad \vee \quad z = \sqrt{2} + \frac{\sqrt{2}}{4}i}$$