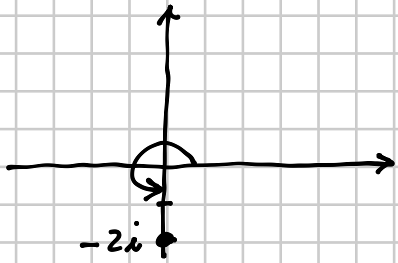
TRASFORMARE IN FORMA TRIG.

256 $-2i$ $\left[2\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)\right]$



$z = x + iy \qquad \rho = \sqrt{x^2+y^2}$

$\rho = 2 \qquad \tan\vartheta = \frac{y}{x}$

$\downarrow$

DATO CHE $x = 0$, NON È DEFINITA LA TANGENTE DI ϑ $\Rightarrow$ IN QUESTO CASO $\vartheta = \frac{3}{2}\pi$

$z = -2i = 2\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$

In generale se $\tan\vartheta$ non è definita, si ha che $\underset{\text{caso } \operatorname{Im}(z)>0}{\vartheta = \frac{\pi}{2}}$ o $\underset{\text{caso } \operatorname{Im}(z)<0}{\vartheta = \frac{3}{2}\pi}$

276 $z_1 = \frac{\sqrt{2}}{2}\left(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\right), \qquad z_2 = \sqrt{3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$

$$z_1 \cdot z_2 = \frac{\sqrt{2}}{2}\cdot\sqrt{3}\left(\cos\left(\frac{11}{6}\pi + \frac{\pi}{4}\right) + i\sin\left(\frac{11}{6}\pi + \frac{\pi}{4}\right)\right) =$$

$$= \frac{\sqrt{6}}{2}\left(\cos\frac{22+3}{12}\pi + i\sin\frac{25}{12}\pi\right) = (*)$$

$$\frac{25}{12}\pi = \frac{24}{12}\pi + \frac{\pi}{12} = 2\pi + \frac{\pi}{12} \qquad \cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} \qquad \sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$(*) = \frac{\sqrt{6}}{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) = \frac{\sqrt{6}}{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}\right) =$$

$$= \frac{6+\sqrt{12}}{8} + i\,\frac{6-\sqrt{12}}{8} = \frac{6+2\sqrt{3}}{8} + i\,\frac{6-2\sqrt{3}}{8} =$$

$$= \boxed{\frac{3+\sqrt{3}}{4} + \frac{3-\sqrt{3}}{4}\,i}$$

287 $(\sqrt{3} - i)(1 + \sqrt{3}\,i)$ $[2\sqrt{3} + 2i]$

IN FORMA ALGEBRICA:

$$z_1 \cdot z_2 = (\sqrt{3} - i)(1 + \sqrt{3}\,i) = \sqrt{3} + 3i - i + \sqrt{3} = \boxed{2\sqrt{3} + 2i}$$

4° QUADR.

$$z_1 = \sqrt{3} - i = 2\left(\cos \frac{11}{6}\pi + i \sin \frac{11}{6}\pi\right)$$

$$\rho_1 = \sqrt{3 + 1} = 2$$

$$\tan \vartheta_1 = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\vartheta_1 = 2\pi - \frac{\pi}{6} = \frac{11}{6}\pi$$

troviamo l'angolo tra 0 e 2π

1° QUADR.

$$z_2 = 1 + \sqrt{3}\,i = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$\rho_2 = \sqrt{1 + 3} = 2$$

$$\tan \vartheta_2 = \sqrt{3}$$

$$\vartheta_2 = \frac{\pi}{3}$$

$$z_1 \cdot z_2 = 2 \cdot 2 \left(\cos\left(\frac{11}{6}\pi + \frac{\pi}{3}\right) + i \sin\left(\frac{11}{6}\pi + \frac{\pi}{3}\right)\right) =$$

$$= 4\left(\cos \frac{13}{6}\pi + i \sin \frac{13}{6}\pi\right) = 4\left(\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right) =$$

$$\frac{13}{6}\pi = 2\pi + \frac{\pi}{6}$$

$$= \boxed{2\sqrt{3} + 2i}$$

STESSO RISULTATO DI PRIMA

307 $\left[\frac{1}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^6 =$ $\left[-\frac{1}{64}i\right]$

$$=\left(\frac{1}{2}\right)^6\left[\cos\left(6\cdot\frac{\pi}{4}\right)+i\sin\left(6\cdot\frac{\pi}{4}\right)\right]=$$

$$=\frac{1}{64}\left[\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi\right]=\frac{1}{64}\left[0+i\cdot(-1)\right]=\boxed{-\frac{1}{64}i}$$

324 $\left[\sqrt{3}\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)\right]^{-3} =$ $\left[\frac{\sqrt{3}}{9}\right]$

$$=\left(\sqrt{3}\right)^{-3}\left(\cos\left(-3\cdot\frac{2}{3}\pi\right)+i\sin\left(-3\cdot\frac{2}{3}\pi\right)\right)=$$

$$=\frac{1}{\left(\sqrt{3}\right)^3}\left(\cos 2\pi-i\sin 2\pi\right)=\frac{1}{3\sqrt{3}}\cdot\left(1-i\cdot 0\right)=\frac{1}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\boxed{\frac{\sqrt{3}}{9}}$$

CALCOLARE IL QUOZIENTE

292 $z_1=4\left(\cos\frac{9}{16}\pi+i\sin\frac{9}{16}\pi\right)$, $z_2=2\left(\cos\frac{5}{16}\pi+i\sin\frac{5}{16}\pi\right)$.

$$\frac{z_1}{z_2}=\frac{4}{2}\left(\cos\left(\frac{9}{16}\pi-\frac{5}{16}\pi\right)+i\sin\left(\frac{9}{16}\pi-\frac{5}{16}\pi\right)\right)=$$

$$=2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=\boxed{\sqrt{2}+\sqrt{2}\,i}$$

334 $z_1^2 + z_2$; $z_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$, $z_2 = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi$.

$$z_1^2 + z_2 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2 + \left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right) =$$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2}{3}\pi\right) + \left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right) =$$

$$= -\frac{1}{2} + \cancel{i\frac{\sqrt{3}}{2}} + \left(-\frac{1}{2} - \cancel{\frac{\sqrt{3}}{2}i}\right) = -1$$

332 $\left[\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^2 \cdot \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) =$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \cdot \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) =$$

$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \boxed{1 + \sqrt{3}\,i}$$