333 $$\frac{\left[\sqrt[8]{2}\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)\right]^8\left[\sqrt[9]{2}\left(\cos\frac{\pi}{36}+i\sin\frac{\pi}{36}\right)\right]^9}{\left[\sqrt[5]{2}\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\right]^5}=$$

$$=\frac{\left(\sqrt[8]{2}\right)^8\left(\cos\frac{8\pi}{8}+i\sin\frac{8\pi}{8}\right)\left(\sqrt[9]{2}\right)^9\left(\cos\frac{9\pi}{36}+i\sin\frac{9\pi}{36}\right)}{\left(\sqrt[5]{2}\right)^5\left(\cos\frac{5\pi}{10}+i\sin\frac{5\pi}{10}\right)}=$$

$$=\frac{\cancel{2}\left(\cos\pi+i\sin\pi\right)\cdot 2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}{\cancel{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}=$$

$$=\frac{(-1)\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)}{i}=\frac{-\sqrt{2}-i\sqrt{2}}{i}\cdot\frac{i}{i}=\frac{-\sqrt{2}i+\sqrt{2}}{-1}=$$

$$=\boxed{-\sqrt{2}+\sqrt{2}i}$$

346 $$\frac{(1+i)^4\cdot(\sqrt{3}-i)^3}{(1+i\sqrt{3})^8}=(*)\qquad\left[\frac{1}{16}(\sqrt{3}-i)\right]$$

$1+i \Rightarrow \rho=\sqrt{1^2+1^2}=\sqrt{2}\quad \tan\vartheta=1\quad \vartheta=\frac{\pi}{4}\quad 1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\sqrt{3}-i \Rightarrow \rho=\sqrt{3+1}=2\quad \tan\vartheta=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}\quad \vartheta=2\pi-\frac{\pi}{6}=\frac{11}{6}\pi$

$\sqrt{3}-i=2\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)$

$1+\sqrt{3}i \Rightarrow \rho=\sqrt{1+3}=2\quad \tan\vartheta=\sqrt{3}\quad \vartheta=\frac{\pi}{3}\quad 1+\sqrt{3}i=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$$(*)=\frac{\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^4\cdot\left[2\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)\right]^3}{\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^8}=$$

$$= \frac{\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^4 \cdot \left[2\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)\right]^3}{\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^8} =$$

$$= \frac{\cancel{2}^{2}\left(\cos\pi+i\sin\pi\right)\cdot \cancel{2}^{3}\left(\overbrace{\cos\frac{11}{2}\pi}^{\cos\left(-\frac{\pi}{2}\right)}+i\,\overbrace{\sin\frac{11}{2}\pi}^{\sin\left(-\frac{\pi}{2}\right)}\right)}{2^{\cancel{8}\,3}\left(\cos\frac{8\pi}{3}+i\sin\frac{8\pi}{3}\right)} = \qquad \frac{8}{3}\pi = 2\pi+\frac{2}{3}\pi$$

$$= \frac{-1\cdot(0-i)}{8\cdot\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)} = \frac{i}{-4+4\sqrt{3}i}\cdot\frac{-4-4\sqrt{3}i}{-4-4\sqrt{3}i} = \frac{-4i+4\sqrt{3}}{(-4)^2+(4\sqrt{3})^2} =$$

$$= \frac{4(\sqrt{3}-i)}{16+16\cdot 3} = \frac{\cancel{4}(\sqrt{3}-i)}{16\cancel{(1+3)}} = \frac{\sqrt{3}-i}{16}$$

341 $\frac{1}{z_1}+\frac{1}{z_2}$; $\quad z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, $\quad z_2=\frac{\sqrt{3}}{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$. $\quad\left[\frac{1+\sqrt{3}}{2}-2i\right]$

$$\frac{1}{z_1}+\frac{1}{z_2} = z_1^{-1}+z_2^{-1} = \left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{-1} + \left[\frac{\sqrt{3}}{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^{-1} =$$

$$= (\sqrt{2})^{-1}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right) + \left(\frac{\sqrt{3}}{3}\right)^{-1}\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right) =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) + \frac{3}{\sqrt{3}}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) =$$

$$= \frac{1}{2}-\frac{1}{2}i+\frac{3}{2\sqrt{3}}-\frac{3}{2}i = \frac{\sqrt{3}+3}{2\sqrt{3}}-2i = \frac{\sqrt{3}+3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}-2i =$$

$$= \frac{3+3\sqrt{3}}{6}-2i = \boxed{\frac{1+\sqrt{3}}{2}-2i}$$