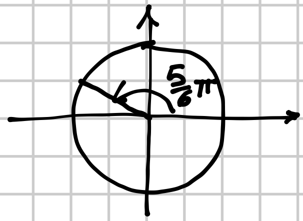
371 $\sqrt{\dfrac{i}{1-\sqrt{3}\,i}}$

Trovare le 2 radici quadrate del numero $z = \dfrac{i}{1-\sqrt{3}i}$

$$z = \frac{i}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{i-\sqrt{3}}{1+3} = -\frac{\sqrt{3}}{4} + \frac{1}{4}i$$

$$\rho = \sqrt{\left(-\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16}+\frac{1}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$z = \frac{1}{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \rho\left(\cos\vartheta + i\sin\vartheta\right)$$



$$= \frac{1}{2}\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$$

FORMULA:

$$z_k = \rho^{\frac{1}{n}}\left(\cos\frac{\vartheta+2k\pi}{n} + i\sin\frac{\vartheta+2k\pi}{n}\right)$$

$k = 0, 1, \dots, n-1$

$$z_0 = \sqrt{\frac{1}{2}}\left(\cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi\right) =$$

$$\cos\frac{5}{12}\pi = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin\frac{5}{12}\pi = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{6}-\sqrt{2}}{4} + i\,\frac{\sqrt{6}+\sqrt{2}}{4}\right) =$$

$$= \frac{\sqrt{3}-1}{4} + i\,\frac{\sqrt{3}-1}{4}$$

$$z_1 = \sqrt{\frac{1}{2}}\left(\cos\frac{\frac{5}{6}\pi+2\pi}{2} + i\sin\frac{\frac{5}{6}\pi+2\pi}{2}\right) = \frac{1}{\sqrt{2}}\left(\cos\frac{17}{12}\pi + i\sin\frac{17}{12}\pi\right) =$$

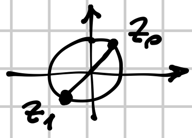
$$\cos\frac{17}{12}\pi = \cos\left(\pi + \frac{5}{12}\pi\right) = -\cos\frac{5}{12}\pi$$

$$\sin\frac{17}{12}\pi = \sin\left(\pi + \frac{5}{12}\pi\right) = -\sin\frac{5}{12}\pi$$

$$= \frac{1}{\sqrt{2}}\left(-\frac{\sqrt{6}-\sqrt{2}}{4} - i\,\frac{\sqrt{6}+\sqrt{2}}{4}\right)$$

$$= -\frac{\sqrt{3}-1}{4} - i\,\frac{\sqrt{3}-1}{4}$$

Infatti le 2 radici quadrate sono gli estremi del segmento



391 $x^4 + 6x^2 + 25 = 0$ $\quad [\pm(1+2i), \pm(1-2i)]$

BIQUADRATICA

$t = x^2$

$t^2 + 6t + 25 = 0$ $\qquad \frac{\Delta}{4} = 9 - 25 = -16 < 0$

le 2 radici quadrate di -16 sono $\pm 4i$

$t = -3 \pm 4i$

$x^2 = -3 \pm 4i$

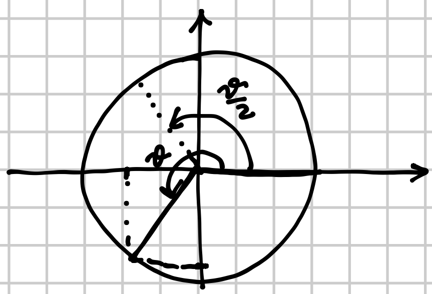
$x^2 = -3-4i \quad \vee \quad x^2 = -3+4i$

devo ancora calcolare le radici quadrate di $-3-4i$ e $-3+4i$

$z = -3-4i = \qquad \rho = \sqrt{9+16} = 5$

$= 5\left(-\frac{3}{5} + i\left(-\frac{4}{5}\right)\right) = \qquad \begin{cases} \cos\vartheta = -\frac{3}{5} \\ \sin\vartheta = -\frac{4}{5} \end{cases}$

$= 5(\cos\vartheta + i \sin\vartheta)$



$z_0 = \sqrt{5}\left(\cos\frac{\vartheta}{2} + i \sin\frac{\vartheta}{2}\right) =$

$= \sqrt{5}\left(-\frac{1}{\sqrt{5}} + i\frac{2}{\sqrt{5}}\right) = -1 + 2i$

$z_1 = 1 - 2i$ (opposto di z_0)

$\cos\frac{\vartheta}{2} = -\sqrt{\frac{1+\cos\vartheta}{2}} =$

$= -\sqrt{\frac{1-\frac{3}{5}}{2}} = -\sqrt{\frac{1}{5}} =$

$= -\frac{1}{\sqrt{5}}$

$\sin\frac{\vartheta}{2} = +\sqrt{\frac{1-\cos\vartheta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} =$

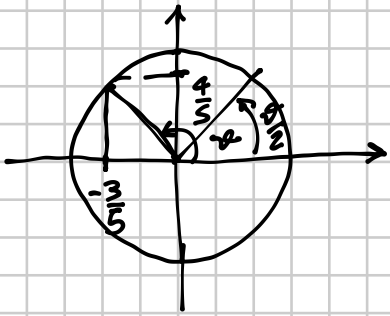
$= \frac{2}{\sqrt{5}}$

$$z = -3 + 4i = \qquad \rho = 5$$

$$= 5\left(-\frac{3}{5} + \frac{4}{5}i\right) =$$

$$= 5\left(\cos\vartheta + i\sin\vartheta\right)$$

$$\begin{cases} \cos\vartheta = -\frac{3}{5} \\ \sin\vartheta = \frac{4}{5} \end{cases}$$



$$\cos\frac{\vartheta}{2} = +\sqrt{\frac{1+\cos\vartheta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \frac{1}{\sqrt{5}}$$

$$\sin\frac{\vartheta}{2} = +\sqrt{\frac{1-\cos\vartheta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \frac{2}{\sqrt{5}}$$

$$z_2 = \sqrt{5}\left(\cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2}\right) =$$

$$= \sqrt{5}\left(\frac{1}{\sqrt{5}} + i\,\frac{2}{\sqrt{5}}\right) =$$

$$= 1 + 2i$$

$z_3 = -1 - 2i$ (opposto di z_2)

Le soluzioni dell'equazione sono $-1+2i$, $1-2i$, $1+2i$, $-1-2i$