417 $\sqrt{3}\,e^{i\frac{\pi}{3}}$ $\left[\frac{\sqrt{3}}{2}+\frac{3}{2}i\right]$ TRASFORMARE IN FORMA ALGEBRICA

$$e^{x+iy}=e^{x}\cdot e^{iy}=e^{x}(\cos y+i\sin y)$$

$$\sqrt{3}\,e^{i\frac{\pi}{3}}=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)=\sqrt{3}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{2}+\frac{3}{2}i$$

424 $24e^{i\frac{\pi}{6}}$ $[12\sqrt{3}+12i]$

$$24e^{i\frac{\pi}{6}}=24\cdot\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)=24\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)=12\sqrt{3}+12i$$

430 $e^{1-\frac{\pi}{2}i}$ $[-ie]$

$$e^{1-\frac{\pi}{2}i}=e^{1}\cdot e^{-\frac{\pi}{2}i}=e\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)=$$

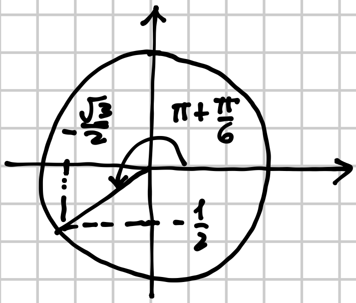
$$=e(0-i)=-ei$$

436 $-\sqrt{3}-i$ $\left[2e^{i\frac{7}{6}\pi}\right]$

SCRIVERE IN FORMA ESPONENZIALE $z=\rho e^{i\vartheta}$

$$\rho=\sqrt{(-\sqrt{3})^2+(-1)^2}=\sqrt{3+1}=2$$

$$z=-\sqrt{3}-i=2\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\Rightarrow\begin{cases}\cos\vartheta=-\frac{\sqrt{3}}{2}\\ \sin\vartheta=-\frac{1}{2}\end{cases}$$



$$z=2e^{i\frac{7}{6}\pi}$$

$$\vartheta=\pi+\frac{\pi}{6}=\frac{7}{6}\pi$$

465 $x^2 - 2x + 2 = 0$ $\left[\sqrt{2}\,e^{\frac{\pi}{4}i}, \sqrt{2}\,e^{\frac{7}{4}\pi i}\right]$

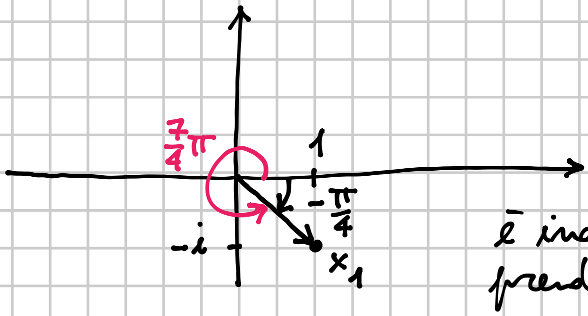
RISOLVERE E SCRIVERE LE SOLUZIONI IN FORMA ESPONENZIALE

$x^2 - 2x + 2 = 0$ $\frac{\Delta}{4} = 1 - 2 = -1 < 0$ IMPOSSIBILE IN $\mathbb{R}$

$x = 1 \pm r$

r = una qualsiasi delle radici quadrate di $\frac{\Delta}{4}$, cioè di -1 $\Rightarrow r = i$

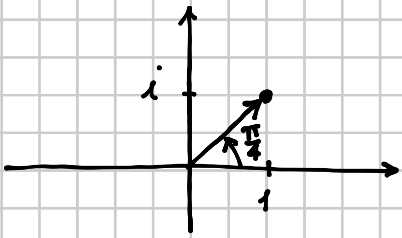
$\boxed{x = 1 - i \quad \vee \quad x = 1 + i}$



$x_1 = 1 - i$ $|x_1| = \sqrt{1+1} = \sqrt{2}$

$x_1 = \sqrt{2}\,e^{-\frac{\pi}{4}i} = \sqrt{2}\,e^{\frac{7}{4}\pi i}$

è indifferente prendere $\vartheta = \frac{7}{4}\pi$ oppure $\vartheta = -\frac{\pi}{4}$



$x_2 = 1 + i$ $|x_2| = \sqrt{2}$

$x_2 = \sqrt{2}\,e^{\frac{\pi}{4}i}$

29 $(3-2i)^3+\dfrac{1-i^{21}}{-1+i^{19}}-\dfrac{5(2+i)}{2-i}-(2+i)(2-i)=$

Calcolare il valore di questa espressione

$$=27+3\cdot 9\cdot(-2i)+3\cdot 3\cdot(-2i)^2+(-2i)^3+\frac{1-i}{-1-i}-\frac{5(2+i)}{2-i}\cdot\frac{2+i}{2+i}-\left(2^2-i^2\right)=$$

A PARTE

$$i^{21}=i^{20}\cdot i=(i^4)^5\cdot i=(1)^5\cdot i=i$$

$$i^{19}=(i^4)^4\cdot i^3=1\cdot i^3=-i$$

$$=27-54i+9\cdot 4\cdot(-1)-8(-i)+\frac{1-i}{-1-i}\cdot\frac{-1+i}{-1+i}-\frac{\cancel{5}(2+i)^2}{\cancel{4+1}}-4-1=$$

$$=27-54i-36+8i+\frac{\cancel{-1}+i+i\cancel{+1}}{1+1}-(4+4i-1)-5=$$

$$=-9-46i+i-3-4i-5=\boxed{-17-49i}$$

51 $x^5 + 4x^3 + x^2 + 4 = 0$

$$x^3(x^2+4) + (x^2+4) = 0$$

$$(x^2+4)(x^3+1) = 0$$

$$(x^2+4)(x+1)(x^2-x+1) = 0$$

$$(x-2i)(x+2i)(x+1)(x^2-x+1) = 0$$

$x^2 - x + 1 = 0$ $\qquad \Delta = 1-4 = -3$ $\qquad \left.\begin{array}{l} r_1 = \sqrt{3}\,i \\ r_2 = -\sqrt{3}\,i \end{array}\right\}$ radici quadrate di -3

$$x = \frac{1 \pm \sqrt{3}\,i}{2}$$

$$\boxed{x = \pm 2i \quad \vee \quad x = -1 \quad \vee \quad x = \frac{1 \pm \sqrt{3}\,i}{2}}$$

Il polinomio si può scomporre:

$$(x+1)(x-2i)(x+2i)\left(x - \frac{1+\sqrt{3}\,i}{2}\right)\left(x - \frac{1-\sqrt{3}\,i}{2}\right)$$

coefficiente di grado max è 1