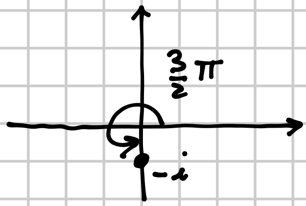
6 Risolvi in $\mathbb{C}$ le equazioni:

....../20 **a.** $x^2+i=0$; **b.** $x^3-27=0$; **c.** $x^2+2\sqrt{2}\,x+5=0$.

a) $x^2+i=0 \qquad x^2=-i$



$z=-i=\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi$

devo trovare le 2 radici quadrate di $-i$

$$z_0=\cos\frac{\frac{3}{2}\pi}{2}+i\sin\frac{\frac{3}{2}\pi}{2}=\cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi=$$

$$=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$$

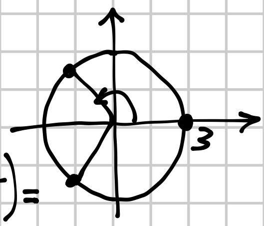
$$z_1=\cos\frac{\frac{3}{2}\pi+2\pi}{2}+i\sin\frac{\frac{3}{2}\pi+2\pi}{2}=$$

$$=\cos\left(\frac{3}{4}\pi+\pi\right)+i\sin\left(\frac{3}{4}\pi+\pi\right)=$$

$$=\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$$

$$\boxed{x=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} \quad \vee \quad x=\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}}$$

b) $x^3-27=0 \qquad z_0=\sqrt[3]{27}=3$



$$z_1=3\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)=$$

$$=3\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=-\frac{3}{2}+i\frac{3}{2}\sqrt{3}$$

$$z_2=3\left(\cos\frac{4}{3}\pi+i\sin\frac{4}{3}\pi\right)=-\frac{3}{2}-i\frac{3}{2}\sqrt{3}$$

$$\boxed{x=3 \quad \vee \quad x=-\frac{3}{2}\pm\frac{3}{2}\sqrt{3}\,i}$$

c) $x^2+2\sqrt{2}\,x+5=0$

$$\frac{\Delta}{4}=2-5=-3$$

$$\boxed{x=-\sqrt{2}\pm\sqrt{3}\,i}$$

6 Risolvi in $\mathbb{C}$ le equazioni:

........./20

a. $x^2-(2-i)x+3-i=0$; **b.** $x^3+x^2+x+1=0$.

a) $x^2-(2-i)x+3-i=0$

$\Delta=(2-i)^2-4(3-i)=$

$=4-1-\cancel{4i}-12+\cancel{4i}=-9$

$r=3i$

una delle 2 radici quadrate di -9

$x=\dfrac{2-i\pm 3i}{2}=$ $\dfrac{2-4i}{2}=1-2i$; $\dfrac{2+2i}{2}=1+i$

$\boxed{x=1-2i \ \vee \ x=1+i}$

b) $x^3+x^2+x+1=0$

$x^2(x+1)+(x+1)=0$

$(x+1)(x^2+1)=0$

$x+1=0 \Rightarrow x=-1$

$x^2+1=0 \Rightarrow x=\pm i$

$\boxed{x=-1 \ \vee \ x=\pm i}$

a. $(2-i)^3 + \dfrac{1+i^4}{1+i^3}$;

Calcolare il valore

$$= 2^3 + 3 \cdot 2^2 \cdot (-i) + 3 \cdot 2 \cdot (-i)^2 + (-i)^3 + \frac{1+(i^2)^2}{1+i^2 \cdot i} =$$

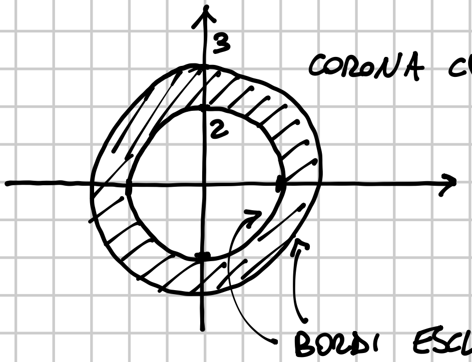
$$= 8 - 12i - 6 + (-1)^3 \cdot \overbrace{i^2}^{-1} \cdot i + \frac{1+1}{1-i} =$$

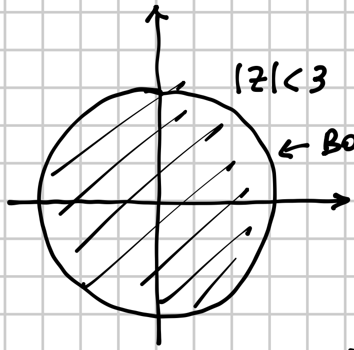
$$= 8 - 12i - 6 + i + \frac{2}{1-i} \cdot \frac{1+i}{1+i} = 2 - 11i + \frac{2+2i}{1+1} =$$

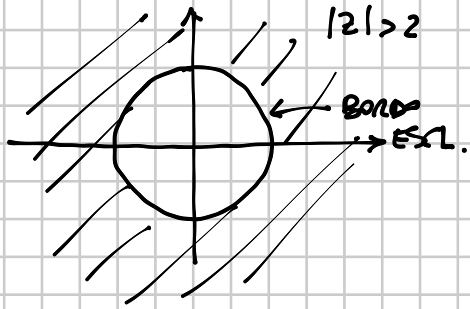
$$= 2 - 11i + \frac{\cancel{2}(1+i)}{\cancel{2}} = 2 - 11i + 1 + i = \boxed{3 - 10i}$$

215 $2 < |z| < 3$ Rappresentare nel piano di Gauss

$$2 < |z| < 3 \iff \begin{cases} |z| < 3 \\ |z| > 2 \end{cases}$$







218 $|z-1| \leq |2-z|$ Rappresentare

$z = x + iy$

$$|x+iy-1| \leq |2-x-iy|$$

$$|(x-1)+iy| \leq |(2-x)-iy|$$

$$\sqrt{(x-1)^2+y^2} \leq \sqrt{(2-x)^2+y^2}$$

$$(x-1)^2+y^2 \leq (2-x)^2+y^2$$

$$\cancel{x^2}+1-2x+\cancel{y^2} \leq 4+\cancel{x^2}-4x+\cancel{y^2}$$

$$2x \leq 3$$

$$x \leq \frac{3}{2}$$