321 $[(\sqrt{2}-\sqrt{6})+(-\sqrt{2}-\sqrt{6})i]^{12}$ $[-4^{12}]$

$$z=(\sqrt{2}-\sqrt{6})+(-\sqrt{2}-\sqrt{6})i$$

$$\rho=\sqrt{(\sqrt{2}-\sqrt{6})^2+(-\sqrt{2}-\sqrt{6})^2}=\sqrt{2+6-\cancel{2\sqrt{12}}+2+6+\cancel{2\sqrt{12}}}=$$

$$=\sqrt{16}=4$$

$$z=4\left[\frac{\sqrt{2}-\sqrt{6}}{4}+\frac{-\sqrt{2}-\sqrt{6}}{4}i\right]=4\left[\cos\frac{17}{12}\pi+i\sin\frac{17}{12}\pi\right]$$

$$\boxed{\begin{array}{ll}\cos\vartheta=\dfrac{\sqrt{2}-\sqrt{6}}{4} & \cos\left(\pi+\dfrac{5}{12}\pi\right)=-\cos\dfrac{5}{12}\pi=-\dfrac{\sqrt{6}-\sqrt{2}}{4}\\ \sin\vartheta=\dfrac{-\sqrt{2}-\sqrt{6}}{4} & \sin\left(\pi+\dfrac{5}{12}\pi\right)=-\sin\dfrac{5}{12}\pi=-\dfrac{\sqrt{6}+\sqrt{2}}{4}\end{array}}$$

$$z^{12}=4^{12}\left[\cos\frac{17\pi}{12}+i\sin\frac{17\pi}{12}\right]^{12}=$$

$$=4^{12}\left[\cos\left(12\cdot\frac{17}{12}\pi\right)+i\sin\left(12\cdot\frac{17}{12}\pi\right)\right]=$$

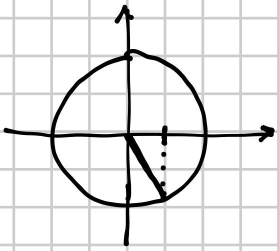
$$=4^{12}\left[\cos 17\pi+i\sin 17\pi\right]=4^{12}\cdot(-1+i\cdot 0)=-4^{12}$$

320 $\left(\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)^4$ $\left[-\frac{81}{2}+\frac{81\sqrt{3}}{2}i\right]$

$$z=\frac{3}{2}-\frac{3\sqrt{3}}{2}i$$

$$|z|=\rho=\sqrt{\left(\frac{3}{2}\right)^2+\left(-\frac{3\sqrt{3}}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{27}{4}}=3$$

$$z=3\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=3\left(\cos\frac{5}{3}\pi+i\sin\frac{5}{3}\pi\right)$$



$$z^4=3^4\left(\cos\frac{20}{3}\pi+i\sin\frac{20}{3}\pi\right)=$$

$$=81\left(\cos\left(6\pi+\frac{2}{3}\pi\right)+i\sin\left(6\pi+\frac{2}{3}\pi\right)\right)=$$

$$=81\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\boxed{-\frac{81}{2}+\frac{81\sqrt{3}}{2}i}$$

347 $\dfrac{\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^4(2i)^4}{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^6} =$ $[-16]$

$$= \frac{\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right)^4 \cdot 16}{\left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right)^6} =$$

$$= \frac{\left(\cos 3\pi + i\sin 3\pi\right)\cdot 16}{\cos 8\pi + i\sin 8\pi} = \frac{-1\cdot 16}{1} = \boxed{-16}$$

450 $\cancel{\sqrt[3]{27e^{\frac{3}{2}\pi i}}}$ $\left[3e^{\frac{\pi}{2}i}, 3e^{\frac{7}{6}\pi i}, 3e^{\frac{11}{6}\pi i}\right]$

↳ Calcolare le radici cubiche di $27e^{\frac{3}{2}\pi i}$

$$z = 27e^{\frac{3}{2}\pi i} = 27\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$$

$$\boxed{e^{\vartheta i} = \cos\vartheta + i\sin\vartheta \text{ DEF.}}$$

$$z_k = \sqrt[3]{27}\left(\cos\frac{\frac{3}{2}\pi + 2k\pi}{3} + i\sin\frac{\frac{3}{2}\pi + 2k\pi}{3}\right)$$

$$k = 0, 1, 2$$

$$z_0 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 3e^{\frac{\pi}{2}i}$$

$$z_1 = 3\left(\cos\frac{\frac{3}{2}\pi + 2\pi}{3} + i\sin\frac{\frac{3}{2}\pi + 2\pi}{3}\right) = 3e^{\frac{7}{6}\pi i}$$

$$z_2 = 3\left(\cos\frac{\frac{3}{2}\pi + 4\pi}{3} + i\sin\frac{\frac{3}{2}\pi + 4\pi}{3}\right) = 3e^{\frac{11}{6}\pi i}$$