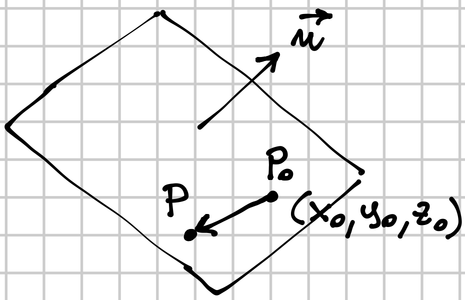
EQUAZIONE GENERALE DI UN PIANO

VETTORE NORMALE

AL PIANO $\vec{n} = (a, b, c)$



PUNTO DEL PIANO $P_0(x_0, y_0, z_0)$

PUNTO GENERICO DEL PIANO $P(x, y, z)$

VETTORE $\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$

Tutti i punti P del piano sono tali che $\overrightarrow{P_0P} \perp \vec{n}$

$$\Updownarrow$$

$$\vec{n} \cdot \overrightarrow{P_0P} = 0 \quad \text{(prodotto scalare)}$$

$$(a, b, c) \cdot (x - x_0, y - y_0, z - z_0) = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax - ax_0 + by - by_0 + cz - cz_0 = 0$$

$$ax + by + cz \underbrace{- ax_0 - by_0 - cz_0}_{d} = 0$$

$$\boxed{ax + by + cz + d = 0}$$

EQUAZIONE GENERALE DEL PIANO

Calcola il prodotto vettoriale tra $\vec{a}$ e $\vec{b}$.

63 $\vec{a}(0; 0; -2)$, $\vec{b}(1; 0; 0)$. $[-2\vec{j}]$

$$\vec{a} \times \vec{b} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 0 & 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right. = -2\vec{j}$$

64 $\vec{a}(1;1;1)$, $\vec{b}(3; 0; -1)$. $[-\vec{i}+4\vec{j}-3\vec{k}]$

$$\vec{a} \times \vec{b} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 0 & -1 & 3 & 0 \end{array}\right. = -\vec{i} + 3\vec{j} - (3\vec{k} - \vec{j}) =$$

$$= -\vec{i} + 3\vec{j} - 3\vec{k} + \vec{j} = -\vec{i} + 4\vec{j} - 3\vec{k}$$

Calcola il prodotto scalare tra $\vec{a}$ e $\vec{b}$.

59 $\vec{a}(1; 0; 1)$, $\vec{b}(4; -1; 7)$. $[11]$

$$\vec{a} \cdot \vec{b} = 1 \cdot 4 + 0 \cdot (-1) + 1 \cdot 7 = 4 + 7 = 11$$

$\quad x_0 \; y_0 \; z_0 \quad a \; b \; c$

99 $B(1; -1; 1)$, $\vec{n}(2; -1; 0)$. $[2x - y - 3 = 0]$

Trovare l'eq. del piano passante per B e con vettore normale $\vec{n}$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$2(x-1) + (-1)(y-(-1)) + 0\cdot(z-1) = 0$$

$$2x - 2 - y - 1 = 0 \qquad \boxed{2x - y - 3 = 0}$$

OPPURE

$$ax + by + cz + d = 0$$ metto le componenti di $\vec{n}(a, b, c)$

$$2x - 1\cdot y + 0\cdot z + d = 0 \qquad 2x - y + d = 0$$

IMPONGO IL PASSAGGIO PER $B(1,-1,1)$

$$2\cdot 1 - (-1) + d = 0$$

$$d = -3$$

$$\boxed{2x - y - 3 = 0}$$

Scrivere l'eq. del piano per A, B, C

122 $A(-1; 0; 3)$, $B(2; 4; 1)$, $C(5; 2; 1)$. $[2x + 3y + 9z - 25 = 0]$

$$ax + by + cz + d = 0$$

$A(-1,0,3) \rightarrow$

$B(2,4,1) \rightarrow$

$C(5,2,1) \rightarrow$

$$\begin{cases} -a + 3c + d = 0 \\ 2a + 4b + c + d = 0 \\ 5a + 2b + c + d = 0 \end{cases} \qquad \begin{cases} a = 3c + d \\ 2(3c + d) + 4b + c + d = 0 \\ 5(3c + d) + 2b + c + d = 0 \end{cases}$$

$$\begin{cases} a = 3c + d \\ 2(3c+d) + 4b + c + d = 0 \\ 5(3c+d) + 2b + c + d = 0 \end{cases} \qquad \begin{cases} // \\ 6c + 2d + 4b + c + d = 0 \\ 15c + 5d + 2b + c + d = 0 \end{cases}$$

$$\begin{cases} // \\ 4b + 7c + 3d = 0 \\ 2b + 16c + 6d = 0 \end{cases} \xrightarrow{:2} \begin{cases} // \\ 4(-8c-3d) + 7c + 3d = 0 \\ b = -8c - 3d \end{cases}$$

$$\begin{cases} // \\ -32c - 12d + 7c + 3d = 0 \\ // \end{cases} \quad \begin{cases} // \\ -25c - 9d = 0 \\ // \end{cases} \quad \begin{cases} a = 3c + d \\ c = -\frac{9}{25} d \\ b = -8c - 3d \end{cases}$$

$$\begin{cases} d = 25 \leftarrow \text{LO DECIDO IO! (BASTA CHE \underline{NON} SIA } d=0) \\ c = -\frac{9}{25} \cdot 25 = -9 \\ a = 3(-9) + 25 = -2 \\ b = -8(-9) - 3 \cdot 25 = +72 - 75 = -3 \end{cases}$$

$$-2x - 3y - 9z + 25 = 0$$

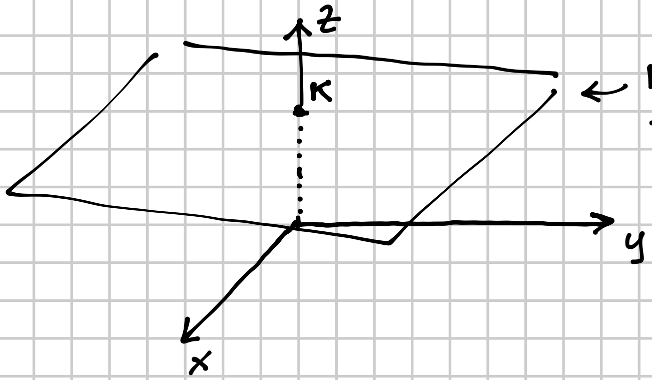
$$\boxed{2x + 3y + 9z - 25 = 0}$$

ALCUNI CASI PARTICOLARI DELL'EQUAZIONE DEL PIANO

$$ax + by + cz + d = 0$$

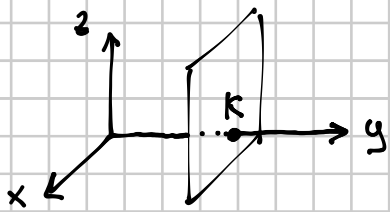
1) $d = 0 \Rightarrow$ il piano passa per $O(0,0,0)$ $\quad ax + by + cz = 0$

2) $a = 0,\ b = 0$ $\quad cz + d = 0 \Rightarrow z = K$ $\quad \left(\text{dove } K = -\frac{d}{c}\right)$

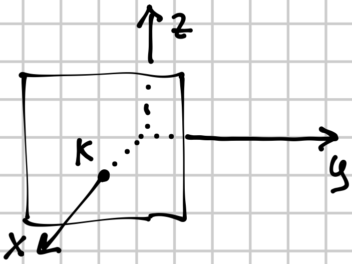


PIANO PARALLELO AL PIANO xy, PASSANTE PER $(0,0,K)$

3) $a = 0,\ c = 0$ $\quad y = K$ $\quad$ PIANO PARALLELO AL PIANO xz



4) $b = 0,\ c = 0$ $\quad x = K$ $\quad$ PIANO PARALLELO AL PIANO yz



5) IN PARTICOLARE:

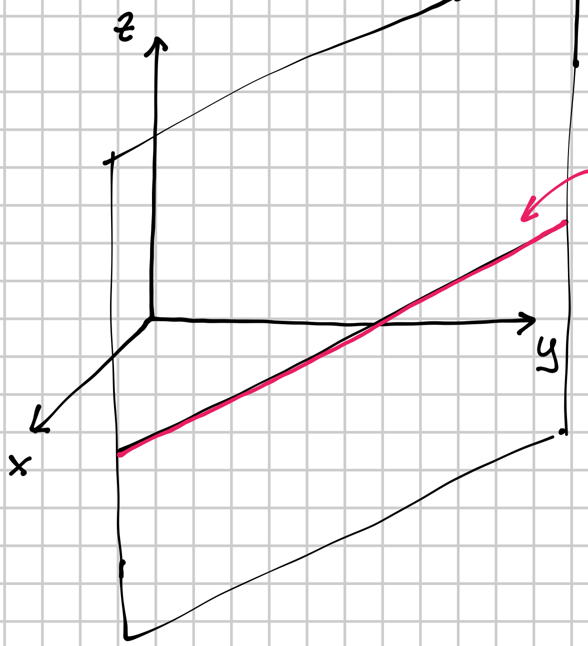
EQ. PIANO xy: $\quad z = 0 \quad (a = 0,\ b = 0,\ d = 0)$

EQ. PIANO xz: $\quad y = 0 \quad (a = 0,\ c = 0,\ d = 0)$

EQ. PIANO yz: $\quad x = 0 \quad (b = 0,\ c = 0,\ d = 0)$

6) $c = 0$ $\qquad ax + by + d = 0$

PIANO PERPENDICOLARE AL PIANO xy
(E PARALLELO ALL'ASSE z)



retta $ax + by + d = 0$
INTERPRETATA nel piano xy

7) $a = 0$ $\qquad by + cz + d = 0$ $\qquad$ PIANO PERPENDICOLARE AL PIANO yz
(PARALLELO ALL'ASSE x)

8) $b = 0$ $\qquad ax + cz + d = 0$ $\qquad$ PIANO PERPENDICOLARE AL PIANO xz
(PARALLELO ALL'ASSE y)