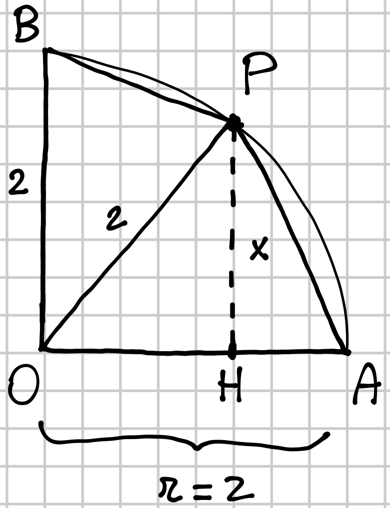
895 Sull'arco AB, quarta parte di una circonferenza di centro O e raggio 2, considera un punto P e la sua proiezione H sul raggio OA. Determina $x = \overline{PH}$ in modo che l'area del quadrilatero $OAPB$ sia maggiore di $\frac{5}{2}$.

$\left[\frac{5-\sqrt{7}}{4} < x < \frac{5+\sqrt{7}}{4}\right]$



LIMITI $0 \leq x \leq 2$

$$\overline{OH} = \sqrt{4 - x^2}$$

$$\overline{AH} = 2 - \sqrt{4-x^2}$$

$$\mathcal{A}_{OAPB}(x) = \mathcal{A}_{OHPB} + \mathcal{A}_{HAP} = \frac{1}{2}(2+x)\sqrt{4-x^2} + \frac{1}{2}\left(2-\sqrt{4-x^2}\right)x$$

$$\begin{cases} 0 \leq x \leq 2 \\ \frac{1}{2}(2+x)\sqrt{4-x^2} + \frac{1}{2}\left(2-\sqrt{4-x^2}\right)x > \frac{5}{2} \end{cases}$$

$$\begin{cases} 0 \leq x \leq 2 \\ (2+x)\sqrt{4-x^2} + 2x - x\sqrt{4-x^2} > 5 \end{cases}$$

$$\begin{cases} 0 \leq x \leq 2 \\ 2\sqrt{4-x^2} + \cancel{x\sqrt{4-x^2}} + 2x - \cancel{x\sqrt{4-x^2}} > 5 \end{cases} \qquad \begin{cases} 0 \leq x \leq 2 \\ 2\sqrt{4-x^2} > 5 - 2x \end{cases}$$

$$\begin{cases} 0 \leq x \leq 2 \\ 5 - 2x < 0 \\ 4 - x^2 \geq 0 \end{cases} \vee \begin{cases} 0 \leq x \leq 2 \\ 5 - 2x \geq 0 \\ 4(4-x^2) > 25 + 4x^2 - 20x \end{cases}$$

$$\begin{cases} 0 \le x \le 2 \\ 5-2x < 0 \\ 4-x^2 \ge 0 \end{cases} \quad \vee \quad \begin{cases} 0 \le x \le 2 \\ 5-2x \ge 0 \\ 4(4-x^2) > 25+4x^2-20x \end{cases}$$

$$\begin{cases} 0 \le x \le 2 \\ x > \frac{5}{2} \\ -2 \le x \le 2 \end{cases} \quad \vee \quad \begin{cases} 0 \le x \le 2 \\ x \le \frac{5}{2} \\ 16-4x^2 > 25+4x^2-20x \end{cases}$$

$$\emptyset$$

$$8x^2-20x+9<0 \qquad \frac{\Delta}{4} = 100-72 = 28$$

$$x = \frac{10 \pm \sqrt{28}}{8} = \frac{10 \pm 2\sqrt{7}}{8} = \frac{5 \pm \sqrt{7}}{4}$$

$$\begin{cases} 0 \le x \le 2 \\ \frac{5-\sqrt{7}}{4} < x < \frac{5+\sqrt{7}}{4} \end{cases}$$

$$\Downarrow$$

$$\boxed{\frac{5-\sqrt{7}}{4} < x < \frac{5+\sqrt{7}}{4}}$$

891

$$\begin{cases} ① \ \dfrac{|x-1|-|x|}{2-\sqrt[3]{x+4}} < 0 \\[2ex] ② \ \dfrac{\sqrt{6-x}-6+4x}{2x-2+\sqrt{9-x}} \le 0 \end{cases}$$

$$\left[\frac{1}{2} < x \le \frac{15}{16}\right]$$

① $$\frac{|x-1|-|x|}{2-\sqrt[3]{x+4}} < 0$$

N] $|x-1|-|x| > 0$ $\qquad |x-1| > |x|$

$$|x-1|^2 > |x|^2$$

$$\cancel{x^2}-2x+1 > \cancel{x^2} \qquad x < \frac{1}{2}$$

D] $2-\sqrt[3]{x+4} > 0$

$$-\sqrt[3]{x+4} > -2$$

elevo al cubo

$$-(x+4) > -8$$

$$-x-4 > -8 \qquad -x > -4 \qquad x < 4$$

		$\frac{1}{2}$		4	
N	+	0	−		−
D	+		+	$\nexists$	−
	+	0	−	$\nexists$	+

$$\frac{1}{2} < x < 4$$

N] ② $\dfrac{\sqrt{6-x}-6+4x}{2x-2+\sqrt{9-x}} \le 0$

D]

N $\quad \sqrt{6-x}-6+4x>0$ $\qquad$ C.E. $\quad 6-x\ge 0 \quad x\le 6$

$\sqrt{6-x} > 6-4x$

$$\begin{cases} 6-4x<0 \\ 6-x\ge 0 \end{cases} \vee \begin{cases} 6-4x\ge 0 \\ 6-x > 36+16x^2-48x \end{cases}$$

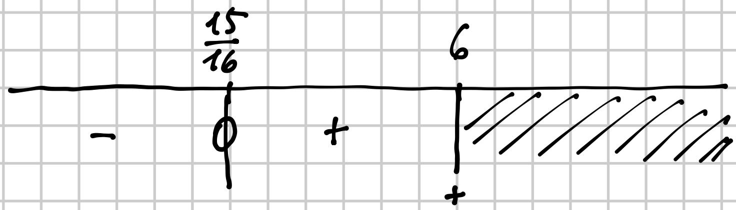
$$\begin{cases} x>\frac{3}{2} \\ x\le 6 \end{cases} \vee \begin{cases} x\le \frac{3}{2} \\ 16x^2-47x+30<0 \end{cases} \qquad \Delta = 2209-1920=289=17^2$$

$$x=\frac{47\pm 17}{32} = \begin{cases} \frac{30}{32}=\frac{15}{16} \\ \frac{64}{32}=2 \end{cases}$$

$$\frac{3}{2}<x\le 6 \quad \vee \quad \begin{cases} x\le \frac{3}{2} \\ \frac{15}{16}<x<2 \end{cases}$$

$\Downarrow$

$$\underbrace{\frac{15}{16}<x\le\frac{3}{2}}_{} \Longrightarrow \quad \frac{15}{16}<x\le 6$$



D) $\quad 2x-2+\sqrt{9-x} > 0 \qquad$ C.E. $\quad 9-x \geq 0 \qquad x \leq 9$

$$\sqrt{9-x} > 2-2x$$

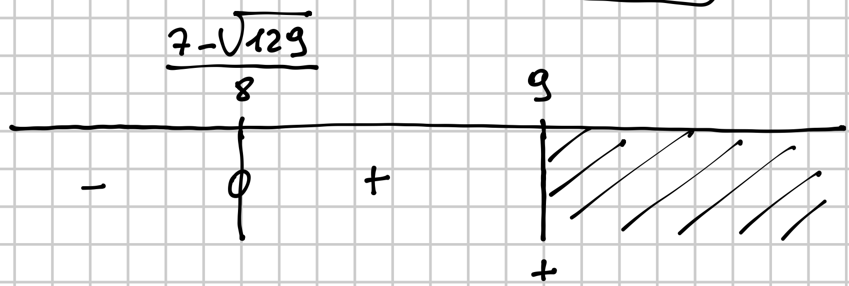
$$\begin{cases} 2-2x < 0 \\ 9-x \geq 0 \end{cases} \quad \vee \quad \begin{cases} 2-2x \geq 0 \\ 9-x > 4+4x^2-8x \end{cases}$$

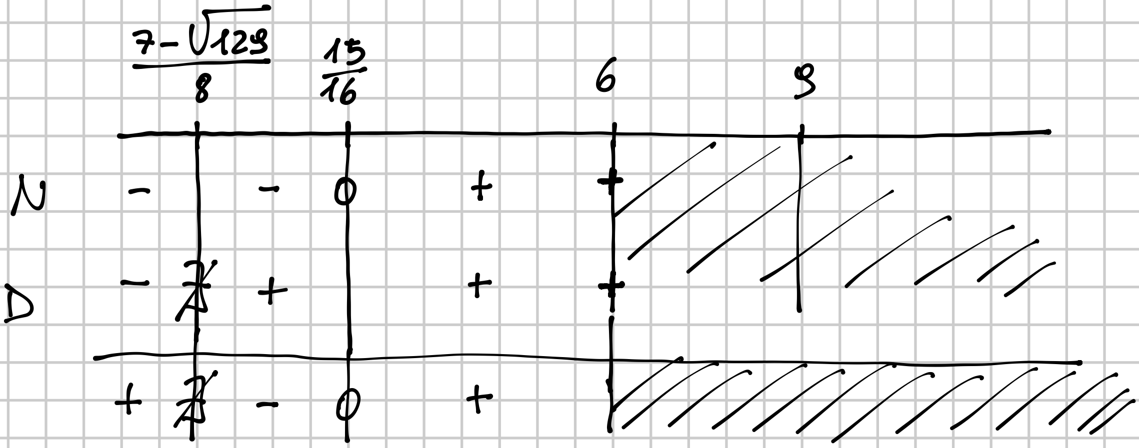
$$\begin{cases} x > 1 \\ x \leq 9 \end{cases} \quad \vee \quad \begin{cases} x \leq 1 \\ 4x^2-7x-5 < 0 \end{cases} \qquad \Delta = 49+80 = 129$$

$$1 < x \leq 9 \qquad \frac{7-\sqrt{129}}{8} < x < \frac{7+\sqrt{129}}{8} \qquad x = \frac{7 \pm \sqrt{129}}{8}$$

$$\Downarrow$$

$$\underbrace{1 < x \leq 9 \quad \vee \quad \frac{7-\sqrt{129}}{8} < x \leq 1} \Longrightarrow \frac{7-\sqrt{129}}{8} < x \leq 9$$

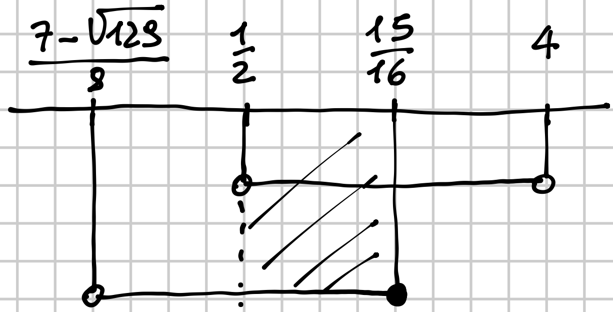




$$\frac{7-\sqrt{129}}{8} < x \leq \frac{15}{16}$$ soluzione della disequazione

Risoluzione del sistema

$$\begin{cases} \frac{1}{2} < x < 4 \\ \\ \frac{7-\sqrt{129}}{8} < x \le \frac{15}{16} \end{cases}$$



$$\boxed{\frac{1}{2} < x \le \frac{15}{16}}$$