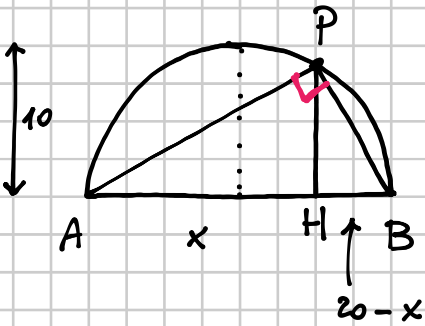
109 Data una semicirconferenza di diametro $\overline{AB} = 20$ cm, trova su di essa un punto P in modo che la sua distanza PH da AB sia minore di 8 cm.

[$0 \le \overline{AH} < 4 \vee 16 < \overline{AH} \le 20$]



$\overline{AB} = 20$ $\qquad 0 \le \overline{PH} \le 10$

$\overline{AH} = x$ $\qquad 0 \le x \le 20$

$\overline{HB} = 20 - x$

II TH. EUCLIDE $\Rightarrow$ $\overline{AH} : \overline{PH} = \overline{PH} : \overline{HB}$

$\overline{PH}^2 = \overline{AH} \cdot \overline{HB}$

$\overline{PH} = \sqrt{\overline{AH} \cdot \overline{HB}} = \sqrt{x(20-x)}$

$\overline{PH} < 8$

$$\begin{cases} \sqrt{x(20-x)} < 8 \\ 0 \le x \le 20 \end{cases} \Rightarrow x(20-x) < 64$$

$20x - x^2 - 64 < 0$

$x^2 - 20x + 64 > 0$ $\qquad \frac{\Delta}{4} = 100 - 64 = 36$

$x = 10 \pm 6 = \begin{cases} 16 \\ 4 \end{cases}$

$x < 4 \vee x > 16$

$$\begin{cases} x < 4 \vee x > 16 \\ 0 \le x \le 20 \end{cases}$$

$$\boxed{0 \le x < 4 \quad \vee \quad 16 < x \le 20}$$

111 Data l'equazione

$$kx^2 - (2k+1)x + k = 0, \quad \text{con } k \neq 0,$$

trova per quali valori di k:

a. le soluzioni sono reali e distinte;

b. non ci sono soluzioni reali.

c. la differenza delle soluzioni è maggiore di 2;

d. il valore assoluto della somma delle soluzioni è minore del loro prodotto.

$\left[\text{a}) k > -\frac{1}{4}; \text{b}) k < -\frac{1}{4}\right.$

$\left.\text{c}) 0 < k < \frac{1+\sqrt{2}}{2}; \text{d}) \nexists k \in \mathbb{R}\right]$

a) $\Delta > 0$

$(2k+1)^2 - 4k^2 > 0 \qquad k \neq 0$

$\cancel{4k^2} + 1 + 4k - \cancel{4k^2} > 0$

$\boxed{k > -\frac{1}{4} \wedge k \neq 0}$

b) $\Delta < 0$

$k < -\frac{1}{4}$

c) $x_1 = \frac{-b-\sqrt{\Delta}}{2a} \qquad x_2 = \frac{-b+\sqrt{\Delta}}{2a}$

SOMMA $x_1 + x_2 = \frac{-b - \cancel{\sqrt{\Delta}} - b + \cancel{\sqrt{\Delta}}}{2a} =$

$= -\frac{2b}{2a} = -\frac{b}{a}$

DIFFERENZA $x_2 - x_1 = \frac{\cancel{-b} + \sqrt{\Delta} + \cancel{b} + \sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{a}$

$\Delta = 1 + 4k$

$\begin{cases} \frac{\sqrt{1+4k}}{k} > 2 \\ k \neq 0 \end{cases} \quad \begin{cases} \frac{\sqrt{1+4k}}{k} - 2 > 0 \\ k \neq 0 \end{cases} \quad \begin{cases} \frac{\sqrt{1+4k} - 2k}{k} > 0 \\ k \neq 0 \end{cases}$

$\frac{\sqrt{1+4k} - 2k}{k} > 0$

C.E. $k \geq -\frac{1}{4}$

N] $\sqrt{1+4k} - 2k > 0$

$\sqrt{1+4k} > 2k$

$\begin{cases} k < 0 \\ 1+4k \geq 0 \end{cases} \vee \begin{cases} k \geq 0 \\ 1+4k > 4k^2 \end{cases}$

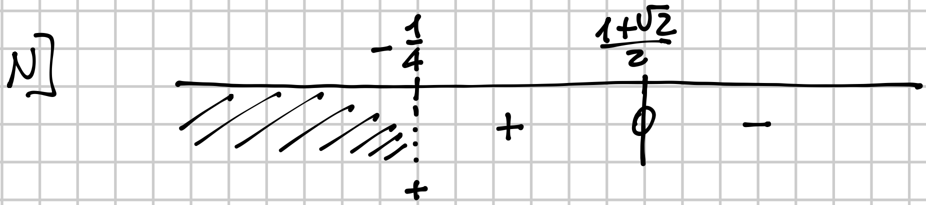
$\begin{cases} k < 0 \\ k \geq -\frac{1}{4} \end{cases} \vee \begin{cases} k \geq 0 \\ 4k^2 - 4k - 1 < 0 \end{cases}$

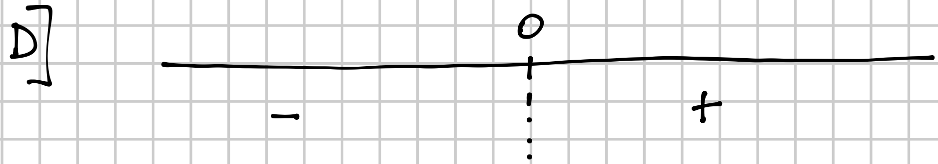
$k = \frac{2 \pm 2\sqrt{2}}{4} = \frac{1-\sqrt{2}}{2}, \ \frac{1+\sqrt{2}}{2}$

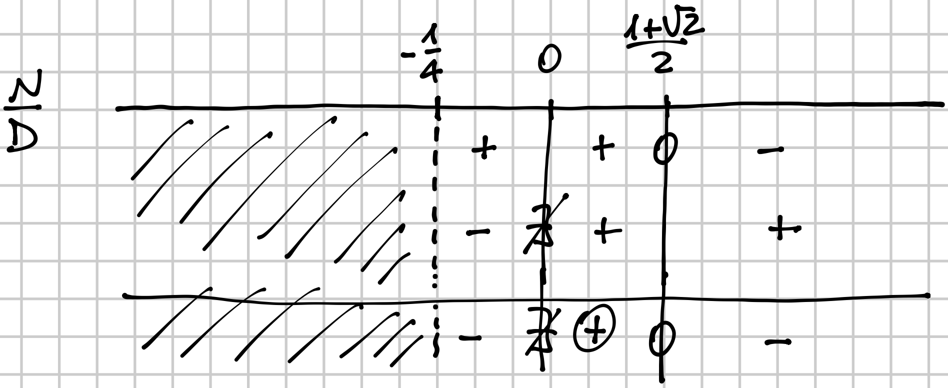
$-\frac{1}{4} \leq k < 0 \quad \vee \begin{cases} k \geq 0 \\ \frac{1-\sqrt{2}}{2} < k < \frac{1+\sqrt{2}}{2} \end{cases}$

$0 \leq k < \frac{1+\sqrt{2}}{2}$

$-\frac{1}{4} \leq k < \frac{1+\sqrt{2}}{2}$







$$0 < K < \frac{1+\sqrt{2}}{2}$$

d) SOMMA $x_1 + x_2 = -\frac{b}{a}$ PRODOTTO $x_1 \cdot x_2 = \frac{c}{a}$

$$\frac{(-b-\sqrt{\Delta})(-b+\sqrt{\Delta})}{4a^2} = \frac{\cancel{b^2} - \cancel{b^2} + 4ac}{4a^2} = \frac{c}{a}$$

$K \geqslant -\frac{1}{4} \wedge K \neq 0$

$$\left| -\frac{b}{a} \right| < \frac{c}{a}$$

$$\left| \frac{b}{a} \right| < \frac{c}{a} \Rightarrow -\frac{c}{a} < \frac{b}{a} < \frac{c}{a}$$

$$\begin{cases} \frac{b}{a} < \frac{c}{a} \\ -\frac{c}{a} < \frac{b}{a} \end{cases} \qquad \begin{cases} \frac{-(2k+1)}{k} < 1 \\ -1 < \frac{-(2k+1)}{k} \end{cases} \qquad \begin{cases} \frac{-2K-1}{K} - 1 < 0 \\ \frac{2K+1}{K} - 1 < 0 \end{cases}$$

con $K \geqslant -\frac{1}{4} \wedge K \neq 0$

$$\begin{cases} \dfrac{-2K-1}{K} - 1 < 0 \\[2ex] \dfrac{2K+1}{K} - 1 < 0 \end{cases} \qquad \begin{cases} \dfrac{-2K-1-K}{K} < 0 \\[2ex] \dfrac{2K+1-K}{K} < 0 \end{cases} \qquad \begin{cases} \dfrac{-3K-1}{K} < 0 \\[2ex] \dfrac{K+1}{K} < 0 \end{cases}$$

$$\begin{cases} \dfrac{3K+1}{K} > 0 \\[2ex] \dfrac{K+1}{K} < 0 \\[2ex] K \geq -\dfrac{1}{4} \wedge K \neq 0 \end{cases}$$

$$\dfrac{3K+1}{K} > 0 \Rightarrow \begin{array}{l} K > -\dfrac{1}{3} \\ K > 0 \end{array}$$

	$-\frac{1}{3}$		0	
$-$	0	$+$		$+$
$-$		$-$	$\nexists$	$+$
$+$	0	$-$	$\nexists$	$+$

$K < -\frac{1}{3} \vee K > 0$

$$\dfrac{K+1}{K} < 0 \Rightarrow \begin{array}{l} K > -1 \\ K > 0 \end{array}$$

	-1		0	
$-$	0	$+$		$+$
$-$		$-$	$\nexists$	$+$
$+$	0	$-$	$\nexists$	$+$

$-1 < K < 0$

$$\begin{cases} K < -\dfrac{1}{3} \vee K > 0 \\[1ex] -1 < K < 0 \\[1ex] K \geq -\dfrac{1}{4} \wedge K \neq 0 \end{cases}$$

IMPOSSIBILE

$$\boxed{\nexists K \in \mathbb{R}}$$