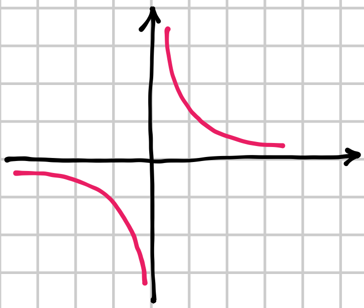
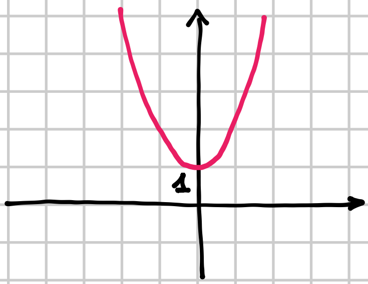
COMPORRE LE FUNZIONI, SPECIFICANDO I DOMINI

284 $f(x) = \frac{1}{x}$; $g(x) = x^2 + 1$. $\left[(f \circ g)(x) = \frac{1}{x^2+1}; (g \circ f)(x) = \frac{1}{x^2} + 1\right]$

$f: \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$ $g: \mathbb{R} \to [1, +\infty)$





$g \circ f: \mathbb{R} \setminus \{0\} \to [1, +\infty)$

DEVO CONTROLLARE CHE L'INSIEME IMMAGINE DI f SIA INCLUSO NEL DOMINIO DI g

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 1$$

$$= \frac{1}{x^2} + 1 = \frac{1 + x^2}{x^2}$$

$f \circ g: \mathbb{R} \to \mathbb{R} \setminus \{0\}$

l'insieme immagine di g è incluso nel dominio di f

$[1, +\infty) \subseteq \mathbb{R} \setminus \{0\}$

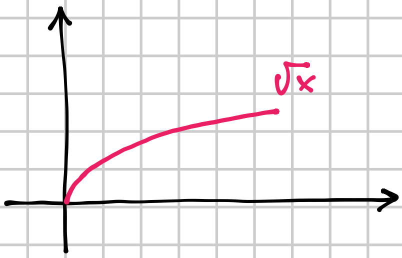
$$(f \circ g)(x) = f(g(x)) =$$

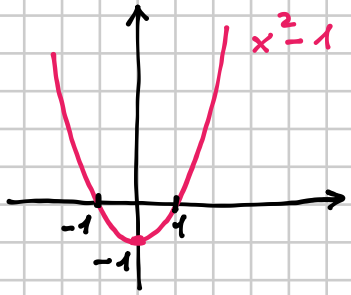
$$= f(x^2 + 1) = \frac{1}{x^2 + 1}$$

COMPORRE

285 $f(x) = \sqrt{x}$; $g(x) = x^2 - 1$. $[(f \circ g)(x) = \sqrt{x^2 - 1}; (g \circ f)(x) = x - 1]$

$f : [0, +\infty) \to [0, +\infty)$ $g : \mathbb{R} \to [-1, +\infty)$





$g \circ f : [0, +\infty) \to [-1, +\infty)$ $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1$

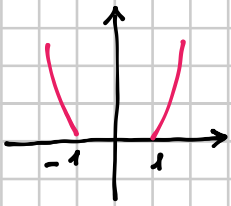
$= x - 1$

$f \circ g$ NON SI PUÓ FARE perché $\operatorname{im} g \not\subseteq \operatorname{dom} f$

Per rendere possibile la composizione dobbiamo modificare il dominio di g, in modo che il suo insieme immagine sia incluso in dom f.

$g : (-\infty, -1] \cup [1, +\infty) \to [0, +\infty)$

$f : [0, +\infty) \to [0, +\infty)$



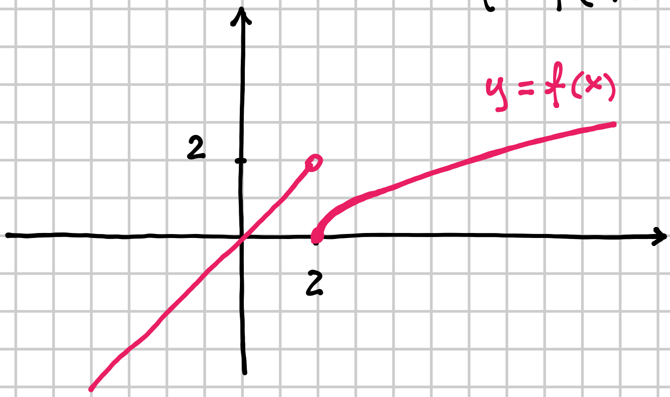
$f \circ g : (-\infty, -1] \cup [1, +\infty) \to [0, +\infty)$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

302 Trova $g \circ f$, con $f(x) = \begin{cases} x & \text{se } x < 2 \\ \sqrt{x-2} & \text{se } x \geq 2 \end{cases}$, $g(x) = \begin{cases} x-2 & \text{se } x < 0 \\ 2x+1 & \text{se } x \geq 0 \end{cases}$. $\left[g \circ f: \begin{cases} 2\sqrt{x-2}+1 & \text{se } x \geq 2 \\ 2x+1 & \text{se } 0 \leq x < 2 \\ x-2 & \text{se } x < 0 \end{cases} \right]$

$f: \mathbb{R} \to \mathbb{R}$ $\qquad g: \mathbb{R} \to \mathbb{R}$ $\qquad g \circ f: \mathbb{R} \to \mathbb{R}$

$$(g \circ f)(x) = g(f(x)) = \begin{cases} f(x) - 2 & \text{se } f(x) < 0 \\ 2f(x) + 1 & \text{se } f(x) \geq 0 \end{cases} =$$



$$= \begin{cases} f(x) - 2 & \text{se } x < 0 \\ 2f(x) + 1 & \text{se } x \geq 0 \end{cases}$$

$$= \begin{cases} x - 2 & \text{se } x < 0 \\ 2x + 1 & \text{se } 0 \leq x < 2 \\ 2\sqrt{x-2} + 1 & \text{se } x \geq 2 \end{cases}$$