106 Dati i punti $A(2k-1; 2)$ e $B(3; 2k+5)$, trova k in modo che il punto medio del segmento AB abbia ascissa doppia dell'ordinata. $[k=-6]$

$$x_M = \frac{2k-1+3}{2} = \frac{2k+2}{2} = \frac{2(k+1)}{2} = k+1$$

$$y_M = \frac{2+2k+5}{2} = \frac{2k+7}{2}$$

$$x_M = 2y_M$$

$$k+1 = \cancel{2}\left(\frac{2k+7}{\cancel{2}}\right)$$

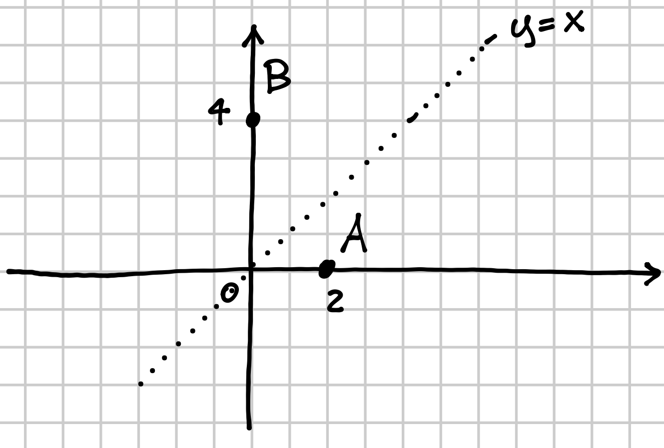
$$k+1 = 2k+7$$

$$\boxed{k=-6}$$

153 Determina le coordinate di un punto P che ha l'ascissa uguale all'ordinata ed è equidistante dai punti $A(2; 0)$ e $B(0; 4)$ e trova il perimetro, l'area e il baricentro del triangolo APB.

$\left[P(3; 3); \sqrt{10}(2+\sqrt{2}); 5; G\left(\frac{5}{3}; \frac{7}{3}\right)\right]$

$A(2,0) \quad B(0,4)$



$P(x,y) \qquad \overline{AP} = \overline{PB}$

$\Downarrow$

$y=x \qquad \sqrt{(x-2)^2+(y-0)^2} = \sqrt{(x-0)^2+(y-4)^2}$

$$\begin{cases} (x-2)^2+y^2 = x^2+(y-4)^2 \\ y=x \end{cases}$$

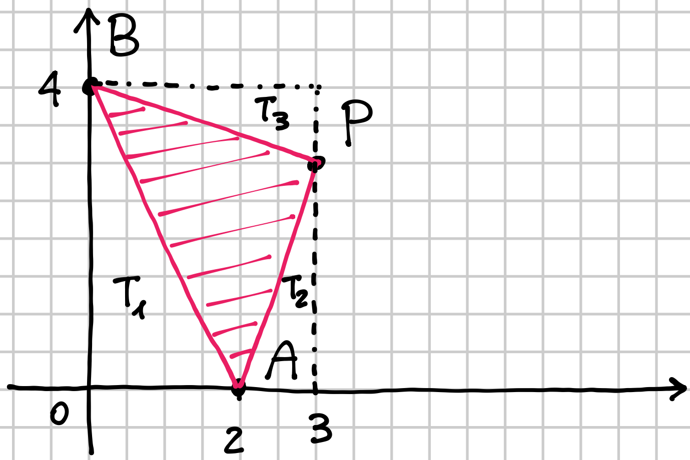
$$\begin{cases} (x-2)^2+y^2 = x^2+(y-4)^2 \\ y = x \end{cases} \qquad \begin{cases} \cancel{x^2}+4-4x+\cancel{y^2} = \cancel{x^2}+\cancel{y^2}+16-8y \\ y = x \end{cases}$$

$$\begin{cases} -4x+8y = 16-4 \\ y = x \end{cases} \qquad \begin{cases} -4x+8x = 12 \\ y = x \end{cases} \qquad \begin{cases} 4x = 12 \\ y = x \end{cases} \qquad \begin{cases} x = 3 \\ y = 3 \end{cases}$$

$P(3,3)$

$A(2,0)$

$B(0,4)$



$$\overline{AB} = \sqrt{(2-0)^2+(0-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$\overline{AP} = \sqrt{(2-3)^2+(0-3)^2} = \sqrt{1+9} = \sqrt{10} = \overline{PB}$$

$$2p = \overline{AB}+\overline{AP}+\overline{PB} = 2\sqrt{5}+2\sqrt{10} = \boxed{2(\sqrt{5}+\sqrt{10})}$$

$$\mathcal{A}_{ABP} = \mathcal{A}_{RETT.} - \mathcal{A}_{T_1} - \mathcal{A}_{T_2} - \mathcal{A}_{T_3} =$$

$$= 4\cdot 3 - \frac{1}{2}\cdot 2\cdot 4 - \frac{1}{2}\cdot 1\cdot 3 - \frac{1}{2}\cdot 1\cdot 3 = 12 - 4 - \frac{3}{2} - \frac{3}{2} = 8-3 = \boxed{5}$$

$$x_G = \frac{x_A+x_B+x_P}{3} = \frac{2+0+3}{3} = \frac{5}{3}$$

$$y_G = \frac{y_A+y_B+y_P}{3} = \frac{0+4+3}{3} = \frac{7}{3}$$

$$\boxed{G\left(\frac{5}{3}, \frac{7}{3}\right)}$$