167 Dati i punti $A(-2; 2)$ e $B(1; 8)$, determina:

a. il punto C di ordinata -1 in modo che il triangolo ABC sia rettangolo in A;

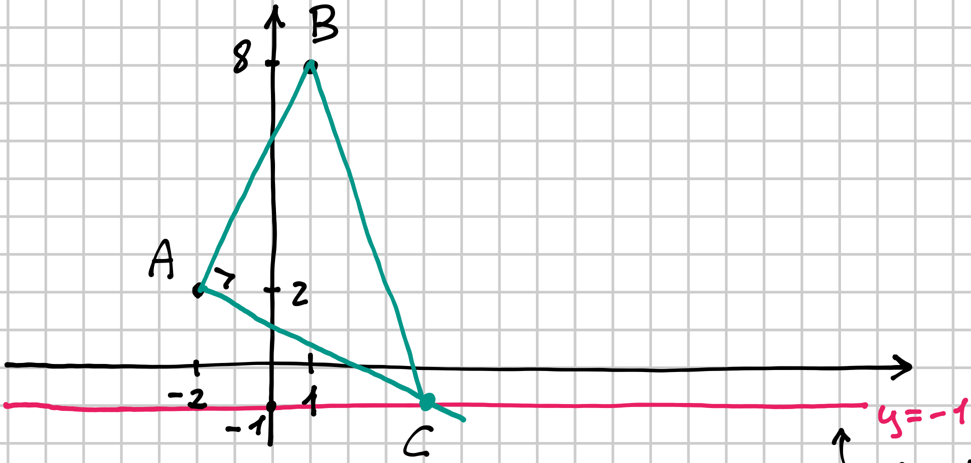
b. il punto D di ascissa -1 in modo che il triangolo ABD sia isoscele con la base su AB;

c. il rapporto tra le aree dei triangoli ABC e ABD;

d. il circocentro dei triangoli ABC e ABD.

$\left[\text{a) } C(4; -1); \text{ b) } D\left(-1; \frac{21}{4}\right); \text{ c) } 12; \text{ d) } P\left(\frac{5}{2}; \frac{7}{2}\right), P'\left(\frac{33}{4}; \frac{5}{8}\right)\right]$

a)



$A(-2,2)$ $B(1,8)$

$C(x,-1)$

il punto C sta su questa retta

Deve essere $\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$

$$\underbrace{(-2-1)^2+(2-8)^2}_{\overline{AB}^2} + \underbrace{(x+2)^2+(-1-2)^2}_{\overline{AC}^2} = \underbrace{(x-1)^2+(-1-8)^2}_{\overline{BC}^2}$$

$$9+36+\cancel{x^2}+4+4x+9 = \cancel{x^2}+1-2x+81$$

$$6x = 82-18-36-4$$

$$6x = 24 \qquad x = 4 \qquad \boxed{C(4,-1)}$$

b) $D(-1,y)$ $\quad \overline{AD} = \overline{BD}$ $\quad A(-2,2)$ $B(1,8)$

$(\overline{AD}^2 = \overline{BD}^2)$

$$(-2+1)^2+(y-2)^2 = (1+1)^2+(8-y)^2$$

$$1+\cancel{y^2}+\cancel{4}-4y = \cancel{4}+64+\cancel{y^2}-16y$$

$$12y = 63 \qquad y = \frac{\cancel{63}^{\,21}}{\cancel{12}_{\,4}} = \frac{21}{4}$$

$$\boxed{D\left(-1,\frac{21}{4}\right)}$$

METODO PER CALCOLARE L'AREA DI UN TRIANGOLO DATI I VERTICI

$A(x_A, y_A)$

$B(x_B, y_B)$

$C(x_C, y_C)$

$$T = \begin{pmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{pmatrix}$$

MATRICE

DETERMINANTE DELLA MATRICE T

$$|T| = \det(T) = \left| \begin{array}{ccc|cc} x_A & y_A & 1 & x_A & y_A \\ x_B & y_B & 1 & x_B & y_B \\ x_C & y_C & 1 & x_C & y_C \end{array} \right| =$$

$$= x_A \cdot y_B \cdot 1 + y_A \cdot 1 \cdot x_C + 1 \cdot x_B \cdot y_C$$

$$- (1 \cdot y_B \cdot x_C + x_A \cdot 1 \cdot y_C + y_A \cdot x_B \cdot 1)$$

$$\mathcal{A}_{ABC} = \frac{1}{2} |\det(T)|$$

MODULO DEL DETERMINANTE

$T_1 : A(-2, 2) \quad B(1, 8) \quad C(4, -1)$

$$\mathcal{A}_{ABC} \quad \left| \begin{array}{ccc|cc} -2 & 2 & 1 & -2 & 2 \\ 1 & 8 & 1 & 1 & 8 \\ 4 & -1 & 1 & 4 & -1 \end{array} \right| = -16 + 8 - 1 - (32 + 2 + 2) =$$

$$= -9 - 36 = -45$$

$$\mathcal{A}_{ABC} = \frac{1}{2} \cdot |-45| = \frac{45}{2}$$

$$\frac{\mathcal{A}_{ABC}}{\mathcal{A}_{ABD}} = \frac{\cancel{45}^{\,3}}{\cancel{2}_{\,1}} \cdot \frac{\cancel{8}^{\,4}}{\cancel{15}_{\,1}} = \boxed{12}$$

$T_2 : A(-2, 2) \quad B(1, 8) \quad D\left(-1, \frac{21}{4}\right)$

$$\left| \begin{array}{ccc|cc} -2 & 2 & 1 & -2 & 2 \\ 1 & 8 & 1 & 1 & 8 \\ -1 & \frac{21}{4} & 1 & -1 & \frac{21}{4} \end{array} \right| = -16 - 2 + \frac{21}{4} - \left(-8 - \frac{21}{2} + 2\right) = -18 + \frac{21}{4} + 6 + \frac{21}{2} =$$

$$= -12 + \frac{63}{4} = \frac{15}{4}$$

$$\mathcal{A}_{ABD} = \frac{1}{2} \cdot \frac{15}{4} = \frac{15}{8}$$

d) Il CIRCOCENTRO di ABC è un punto P_1 tale che

$$\overline{P_1A} = \overline{P_1B} = \overline{P_1C} \qquad A(-2,2) \quad B(1,8) \quad C(4,-1)$$

$P_1(x,y)$

$$\begin{cases} \underbrace{(x+2)^2+(y-2)^2}_{\overline{P_1A}^2} = \underbrace{(x-1)^2+(y-8)^2}_{\overline{P_1B}^2} \\ \underbrace{(x-1)^2+(y-8)^2}_{\overline{P_1B}^2} = \underbrace{(x-4)^2+(y+1)^2}_{\overline{P_1C}^2} \end{cases}$$

$$\begin{cases} \cancel{x^2}+4+4x+\cancel{y^2}+4-4y = \cancel{x^2}+1-2x+\cancel{y^2}+64-16y \\ \cancel{x^2}+\cancel{1}-2x+\cancel{y^2}+64-16y = \cancel{x^2}+16-8x+\cancel{y^2}+\cancel{1}+2y \end{cases}$$

$$\begin{cases} 6x+12y=57 \\ 6x-18y=-48 \end{cases}$$
$$// \quad 30y = 105$$

$$\begin{cases} 6x+42=57 \\ y=\frac{105}{30}=\frac{21}{6}=\frac{7}{2} \end{cases} \quad \begin{cases} 6x=15 \\ y=\frac{7}{2} \end{cases} \quad \begin{cases} x=\frac{5}{2} \\ y=\frac{7}{2} \end{cases}$$

$\boxed{P_1\left(\frac{5}{2},\frac{7}{2}\right)}$ CIRCOCENTRO DI ABC

osserviamo che P_1 è il punto medio dell'ipotenusa del triangolo rettangolo ABC

In modo analogo per trovare il circocentro P_2 di ABD

$A(-2,2)$
$B(1,8)$
$D\left(-1,\frac{21}{4}\right)$

$$\begin{cases} (x+2)^2+(y-2)^2=(x-1)^2+(y-8)^2 \\ (x-1)^2+(y-8)^2=(x+1)^2+\left(y-\frac{21}{4}\right)^2 \end{cases}$$

$P_2(x,y)$

$$\begin{cases} (x+2)^2+(y-2)^2=(x-1)^2+(y-8)^2 \\ (x-1)^2+(y-8)^2=(x+1)^2+\left(y-\frac{21}{4}\right)^2 \end{cases}$$

$$\begin{cases} \cancel{x^2}+4+4x+\cancel{y^2}+4-4y=\cancel{x^2}+1-2x+\cancel{y^2}+64-16y \\ \cancel{x^2}+\cancel{1}-2x+\cancel{y^2}+64-16y=\cancel{x^2}+\cancel{1}+2x+\cancel{y^2}-\frac{21}{2}y+\frac{441}{16} \end{cases}$$

$$\begin{cases} 6x+12y=57 \\ -4x-16y+\frac{21}{2}y=\frac{441}{16}-64 \end{cases} \qquad \begin{cases} 6x+12y=57 \\ -4x-\frac{11}{2}y=-\frac{583}{16} \end{cases}$$

$$\begin{cases} x=\frac{57-12y}{6} \\ -\cancel{4}^{2}\,\frac{57-12y}{\cancel{6}_{3}}-\frac{11}{2}y=-\frac{583}{16} \end{cases} \qquad \frac{-114+24y}{3}-\frac{11}{2}y=-\frac{583}{16}$$

$$\frac{-1824+384y-264y}{\cancel{48}}=\frac{-1749}{\cancel{48}} \qquad 120y=75 \qquad y=\frac{\cancel{75}^{5}}{\cancel{120}_{8}}=\frac{5}{8}$$

$$x=\frac{57-\cancel{12}^{3}\cdot\frac{5}{\cancel{8}_{2}}}{6}=\frac{57-\frac{15}{2}}{6}=$$

$$=\frac{114-15}{2}\cdot\frac{1}{6}=\frac{\cancel{99}^{33}}{\cancel{12}_{4}}=\frac{33}{4}$$

$$\begin{cases} x=\frac{33}{4} \\ y=\frac{5}{8} \end{cases} \qquad \boxed{P_2\left(\frac{33}{4},\frac{5}{8}\right)}$$ CIRCOCENTRO DI ABD