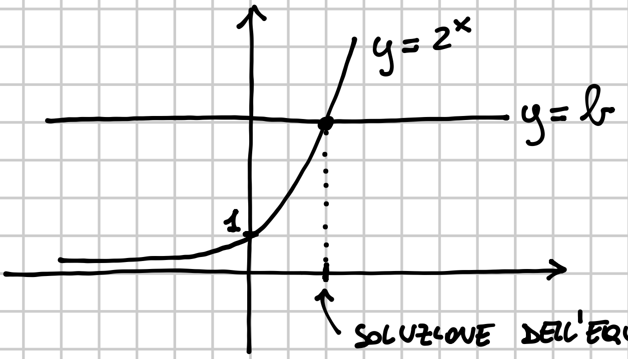
EQUAZIONI ESPONENZIALI

$2^x = b \qquad b > 0$ (se $b \leq 0$ l'eq. è IMPOSSIBILE)



$$\begin{cases} y = 2^x \\ y = b \end{cases}$$

SOLUZIONE DELL'EQUAZIONE (ASCISSA DEL PUNTO DI INTERSEZIONE)
$\downarrow$
esiste se e solo se $b > 0$

$2^x = -4$ è IMPOSSIBILE

$2^x = 0$ è IMPOSSIBILE

$y = 2^x$ è INIETTIVA $\quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$2^x = 8 \qquad \underbrace{2^x}_{f(x)} = \underbrace{2^3}_{f(3)} \Rightarrow x = 3$ perché l'esponenziale $y = 2^x$ è una funzione iniettiva

136 $3^{x+1} = 27$ $\quad [2]$

$$3^{x+1} = 3^3 \Rightarrow x + 1 = 3 \Rightarrow \boxed{x = 2}$$

137 $5^{2x} = \frac{1}{25}$ $\quad [-1]$

$$5^{2x} = 5^{-2} \qquad 2x = -2 \qquad \boxed{x = -1}$$

144 $3^x = \frac{9 \cdot \sqrt{3}}{\sqrt[4]{3}}$ $\quad \left[\frac{9}{4}\right]$

$$3^x = \frac{3^2 \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{4}}} = 3^{2+\frac{1}{2}-\frac{1}{4}} \qquad x = 2 + \frac{1}{2} - \frac{1}{4} = \frac{8+2-1}{4} = \frac{9}{4}$$

151 $4^{x+2} = \overbrace{1 - \sqrt{2}}^{<0}$ [impossibile]

152 $3^x \cdot 27 = 9^{2x}$ [1]

$3^x \cdot 3^3 = (3^2)^{2x}$

$3^{x+3} = 3^{4x}$

$x+3 = 4x \qquad 3x = 3 \qquad x = 1$

165 $\sqrt{27\sqrt{9^x}} = 3^{x-2} \cdot 27$ [1]

$\sqrt{3^3\, 9^{\frac{x}{2}}} = 3^{x-2} \cdot 3^3$

$\sqrt{3^3\, (3^2)^{\frac{x}{2}}} = 3^{x-2+3}$

$\sqrt{3^3 \cdot 3^x} = 3^{x+1}$

$\left(3^{3+x}\right)^{\frac{1}{2}} = 3^{x+1} \qquad 3^{\frac{3+x}{2}} = 3^{x+1} \qquad \frac{3+x}{2} = x+1$

$3+x = 2x+2$

$x = 1$

169 $2^x + 2^{x+1} = -2^{x-1} + 7$ [1]

$$2^x + 2^x \cdot 2 = -2^x \cdot 2^{-1} + 7$$

$$2^x + 2 \cdot 2^x + \frac{1}{2} \cdot 2^x = 7$$

$$2^x \left(1 + 2 + \frac{1}{2}\right) = 7$$

$$2^x \frac{2+4+1}{2} = 7 \qquad 2^x \cdot \frac{\cancel{7}}{2} = \cancel{7} \qquad 2^x = 2 \qquad x = 1$$

186 $2^{x+2} - 4 \cdot 5^{x+2} = 25 \cdot 5^x - 4 \cdot 2^x$ [−3]

$$2^{x+2} + 4 \cdot 2^x = 25 \cdot 5^x + 4 \cdot 5^{x+2}$$

$$2^x \cdot 2^2 + 4 \cdot 2^x = 25 \cdot 5^x + 4 \cdot 5^x \cdot 5^2$$

$$2^x (4 + 4) = 5^x (25 + 4 \cdot 25)$$

$$2^x \cdot 8 = 5^x \cdot 125 \Rightarrow$$

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$$2^x \cdot 2^3 = 5^x \cdot 5^3$$

$$2^{x+3} = 5^{x+3}$$

$$\Downarrow$$

$$x + 3 = 0 \Rightarrow \boxed{x = -3}$$

$\Downarrow$ ALTRO MODO

$$\frac{2^x}{5^x} = \frac{125}{8}$$

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^{-3}$$

$$\Downarrow$$

$$\boxed{x = -3}$$

191 $3^{2x} - 9 \cdot 3^x + 3 = \frac{1}{3} \cdot 3^x$ [− 1; 2]

$$\left(3^x\right)^2 - 9 \cdot 3^x - \frac{1}{3} \cdot 3^x + 3 = 0$$

$$\left(3^x\right)^2 - 3^x\left(9 + \frac{1}{3}\right) + 3 = 0$$

$$\left(3^x\right)^2 - \frac{28}{3} \cdot 3^x + 3 = 0 \qquad 3^x = t$$

$$t^2 - \frac{28}{3}t + 3 = 0$$

$$3t^2 - 28t + 9 = 0 \qquad \frac{\Delta}{4} = 196 - 27 = 169 = 13^2$$

$$t = \frac{14 \pm 13}{3} = \begin{cases} \frac{1}{3} \Rightarrow 3^x = \frac{1}{3} \Rightarrow \\ 9 \Rightarrow 3^x = 9 \Rightarrow \end{cases} \boxed{\begin{matrix} x = -1 \\ \vee \\ x = 2 \end{matrix}}$$

207 $\frac{4}{2^x - 1} + \frac{3}{2^x + 1} = 5$ [1]

C.E.

$$\begin{cases} 2^x - 1 \neq 0 & x \neq 0 \\ 2^x + 1 \neq 0 & \forall x \end{cases}$$

$\Downarrow$

$x \neq 0$

$2^x = t$

$$\frac{4}{t-1} + \frac{3}{t+1} = 5$$

$$\frac{4(t+1) + 3(t-1)}{\cancel{(t-1)(t+1)}} = \frac{5(t^2-1)}{\cancel{(t-1)(t+1)}}$$

$$\Delta = 49 + 120 = 169 = 13^2$$

$$4t + 4 + 3t - 3 = 5t^2 - 5 \qquad 5t^2 - 7t - 6 = 0 \qquad t = \frac{7 \pm 13}{10} = \begin{cases} -\frac{3}{5} \\ 2 \end{cases}$$

$$2^x = -\frac{3}{5} \quad \vee \quad 2^x = 2$$

IMPOSSIBILE $\qquad \Downarrow \boxed{x = 1}$