250 $\left(\frac{2}{5}\right)^{x-1} - \left(\frac{5}{2}\right)^{\frac{x-1}{x}} = 0$ [±1]

C.E. $x \neq 0$

$$\left[\left(\frac{5}{2}\right)^{-1}\right]^{x-1} - \left(\frac{5}{2}\right)^{\frac{x-1}{x}} = 0$$

$$\left(\frac{5}{2}\right)^{-x+1} = \left(\frac{5}{2}\right)^{\frac{x-1}{x}}$$

$$-x+1 = \frac{x-1}{x}$$

$$\frac{-x^2+x}{\cancel{x}} = \frac{x-1}{\cancel{x}}$$

$$-x^2+\cancel{x} = \cancel{x}-1$$

$$x^2 = 1 \qquad \boxed{x = \pm 1}$$

251 $\frac{5^x}{5^x+1} - \frac{1}{25^x-1} = 1$ [impossibile]

C.E.

$25^x - 1 \neq 0 \qquad 25^x \neq 1$

$x \neq 0$

$$\frac{5^x}{5^x+1} - \frac{1}{5^{2x}-1} = 1$$

$t = 5^x$

$$\frac{t}{t+1} - \frac{1}{t^2-1} = 1$$

$(t-1)(t+1)$

$$\frac{t(t-1)-1}{\cancel{(t-1)(t+1)}} = \frac{t^2-1}{\cancel{(t-1)(t+1)}}$$

$$\cancel{t^2} - t - \cancel{1} = \cancel{t^2} - \cancel{1}$$

$t = 0$

$5^x = 0$ IMPOSSIBILE

(perché $5^x > 0$ $\forall x$)

257 $\begin{cases} 3^x + 3^y = 10 \\ 3^{x+1} - 3^y = -6 \end{cases}$ $[(0;2)]$

$$\begin{cases} 3^y = 10 - 3^x \\ 3^{x+1} - (10 - 3^x) = -6 \end{cases} \qquad \begin{cases} 3^y = 10 - 3^x \\ 3 \cdot 3^x - 10 + 3^x = -6 \end{cases}$$

$$3^x(3+1) = -6 + 10$$

$$4 \cdot 3^x = 4$$

$$3^x = 1 \Rightarrow x = 0$$

$$\begin{cases} 3^y = 10 - 3^0 \\ x = 0 \end{cases} \qquad \begin{cases} 3^y = 9 \\ x = 0 \end{cases}$$

$$\begin{cases} 3^y = 3^2 \\ x = 0 \end{cases} \qquad \begin{cases} y = 2 \\ x = 0 \end{cases} \qquad \begin{cases} x = 0 \\ y = 2 \end{cases}$$

262 $\begin{cases} 4^{y^2} - 2^{4x} = 0 \\ \dfrac{625^x \cdot 25^x}{\sqrt{125}} = \sqrt{5}\left(\dfrac{1}{5}\right)^y \end{cases}$ $\left[\left(\frac{1}{2}; -1\right); \left(\frac{2}{9}; \frac{2}{3}\right)\right]$

$$\begin{cases} 2^{2y^2} = 2^{4x} \\ \dfrac{5^{4x} \cdot 5^{2x}}{5^{\frac{3}{2}}} = 5^{\frac{1}{2}} \cdot 5^{-y} \end{cases} \qquad \begin{cases} \cancel{2}y^2 = \cancel{4}^{\,2}x \\ 5^{4x+2x-\frac{3}{2}} = 5^{\frac{1}{2}-y} \end{cases}$$

$$\begin{cases} y^2 = 2x \\ 6x - \frac{3}{2} = \frac{1}{2} - y \end{cases} \qquad \begin{cases} (2-6x)^2 = 2x \Rightarrow 4 + 36x^2 - 24x - 2x = 0 \\ y = 2 - 6x \end{cases}$$

$$36x^2 - 26x + 4 = 0$$

$$18x^2 - 13x + 2 = 0$$

$$\Delta = 169 - 144 = 25$$

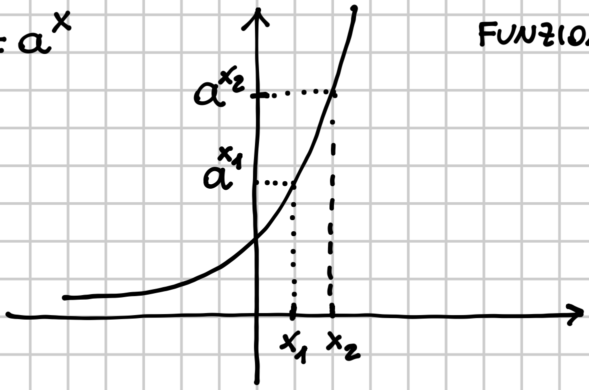
$$x = \frac{13 \pm 5}{36} = \begin{cases} \frac{8}{36} = \frac{2}{9} \\ \frac{18}{36} = \frac{1}{2} \end{cases}$$

$$\boxed{\begin{cases} x = \frac{2}{9} \\ y = 2 - 6 \cdot \frac{2}{9} = 2 - \frac{4}{3} = \frac{2}{3} \end{cases} \vee \begin{cases} x = \frac{1}{2} \\ y = 2 - 3 = -1 \end{cases}}$$

DISEQUAZIONI ESPONENZIALI

$\boxed{\text{CASO} \quad a > 1}$

$y = a^x$



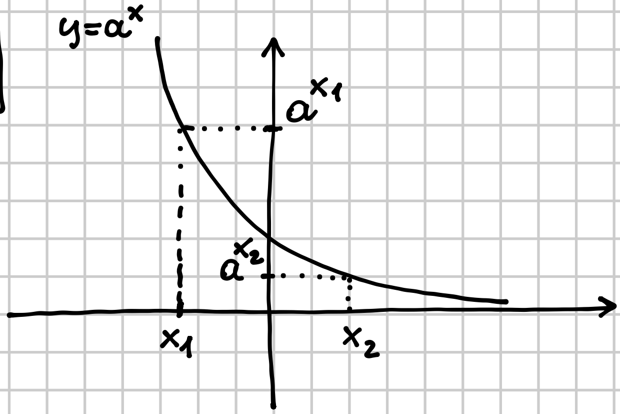
FUNZIONE STRETT. CRESCENTE

$a^{x_1} < a^{x_2} \Rightarrow x_1 < x_2$

ESEMPIO

$2^{x+1} < 2^{3-x} \Rightarrow x+1 < 3-x \qquad 2x < 2 \qquad x < 1$

$\boxed{\text{CASO} \quad 0 < a < 1}$



$a^{x_1} > a^{x_2} \Rightarrow x_1 < x_2$

se $0<a<1$, quando passo agli esponenti
INVERTO la disuguaglianza

ESEMPIO

$\left(\frac{1}{3}\right)^x < \left(\frac{1}{3}\right)^{2x+3} \Rightarrow x > 2x+3$

$x - 2x > 3$

$-x > 3 \Rightarrow x < -3$

269 $\left(\frac{3}{2}\right)^x < \frac{8}{27}$ $[x < -3]$

$$\left(\frac{3}{2}\right)^x < \left(\frac{2}{3}\right)^3$$

$$\left(\frac{3}{2}\right)^x < \left(\frac{3}{2}\right)^{-3} \underset{\frac{3}{2}>1}{\Longrightarrow} x < -3$$

275 $\left(\frac{1}{5}\right)^{2x+1} < 625$ $\left[x > -\frac{5}{2}\right]$

$$\left(\frac{1}{5}\right)^{2x+1} < 5^4$$

$$\left(\frac{1}{5}\right)^{2x+1} < \left(\frac{1}{5}\right)^{-4} \underset{\frac{1}{5}<1}{\Longrightarrow} 2x+1 > -4$$

$$2x > -5 \qquad \boxed{x > -\frac{5}{2}}$$

se avessi "invertito" il 1º membro

$$5^{-2x-1} < 5^4 \underset{5>1}{\Longrightarrow} -2x-1 < 4$$

$$-2x < 5 \qquad x > -\frac{5}{2}$$

288 $34\left(\frac{3}{5}\right)^x < 25\left(\frac{9}{25}\right)^x + 9$ $\qquad [x < 0 \vee x > 2]$

$$34\left(\frac{3}{5}\right)^x < 25\left(\frac{3}{5}\right)^{2x} + 9 \qquad t = \left(\frac{3}{5}\right)^x$$

$$34t < 25t^2 + 9$$

$$25t^2 - 34t + 9 > 0 \qquad \frac{\Delta}{4} = 17^2 - 225 = 289 - 225 = 64$$

$$t = \frac{17 \pm 8}{25} = \begin{cases} \frac{9}{25} \\ 1 \end{cases}$$

$$t < \frac{9}{25} \quad \vee \quad t > 1$$

$$\left(\frac{3}{5}\right)^x < \frac{9}{25} \quad \vee \quad \left(\frac{3}{5}\right)^x > 1$$

$$\left(\frac{3}{5}\right)^x < \left(\frac{3}{5}\right)^2 \quad \vee \quad \left(\frac{3}{5}\right)^x > \left(\frac{3}{5}\right)^0$$

$$x > 2 \quad \vee \quad x < 0 \quad \Rightarrow \quad \boxed{x < 0 \quad \vee \quad x > 2}$$

perché $\frac{3}{5} < 1$