290 $\log_2(\sqrt{5-x^2}-x)=0$ [1]

$$\begin{cases} \sqrt{5-x^2}-x>0 \\ \sqrt{5-x^2}-x=1 \end{cases} \implies \sqrt{5-x^2}-x=1$$

$$\sqrt{5-x^2}=x+1$$

$$\begin{cases} x+1\geq 0 \\ 5-x^2=(x+1)^2 \end{cases} \quad \begin{cases} x\geq -1 \\ 5-x^2=x^2+2x+1 \end{cases}$$

$$\begin{cases} x\geq -1 \\ 2x^2+2x-4=0 \end{cases} \quad \begin{cases} x\geq -1 \\ x^2+x-2=0 \\ (x+2)(x-1)=0 \end{cases} \quad \begin{cases} x\geq -1 \\ \underbrace{x=-2}_{NON\,ACC.}\ \lor\ x=1 \end{cases} \implies \boxed{x=1}$$

326 $\log(2x^2 + 5x - 3) - \log(x + 3) = \log(4 - x)$

C.E. $\begin{cases} 2x^2 + 5x - 3 > 0 \\ x + 3 > 0 \\ 4 - x > 0 \end{cases}$ $\begin{cases} 2x^2 + 6x - x - 3 > 0 \\ x > -3 \\ -x > -4 \end{cases}$ $\begin{cases} 2x(x+3) - (x+3) > 0 \\ x > -3 \\ x < 4 \end{cases}$

$\begin{cases} (x+3)(2x-1) > 0 \\ x > -3 \\ x < 4 \end{cases}$ $\begin{cases} x < -3 \quad \lor \quad x > \frac{1}{2} \\ x > -3 \\ x < 4 \end{cases}$ $\frac{1}{2} < x < 4$

$$\log \frac{2x^2 + 5x - 3}{x + 3} = \log(4 - x)$$

$$\frac{2x^2 + 5x - 3}{x + 3} = 4 - x$$

$$\frac{(\cancel{x+3})(2x-1)}{\cancel{x+3}} = 4 - x$$

$$2x - 1 = 4 - x$$

$$3x = 5 \qquad \boxed{x = \frac{5}{3}} \quad \text{accettabile per C.E.}$$

339 $4(\log_2 x)^2 + 2\log_2 x - 2 = 0$ $\left[\dfrac{1}{2}; \sqrt{2}\right]$ C.E. $x > 0$

$$t = \log_2 x$$

$$4t^2 + 2t - 2 = 0 \qquad 2t^2 + t - 1 = 0 \qquad \Delta = 1 + 8 = 9$$

$$t = \frac{-1 \pm 3}{4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$$t = -1 \quad \lor \quad t = \frac{1}{2}$$

$$\log_2 x = -1 \qquad\qquad \log_2 x = \frac{1}{2}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$x = 2^{-1} = \frac{1}{2} \qquad x = 2^{\frac{1}{2}} = \sqrt{2} \qquad \boxed{x = \frac{1}{2} \quad \lor \quad x = \sqrt{2}}$$

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350 $\log_3 \sqrt{x} \,(\log_3 x + 1) - 2\log_3 x = 2$ $\left[\dfrac{1}{3}; 81\right]$

C.E.
$x > 0$ $\log_3 x^{\frac{1}{2}} \left(\log_3 x + 1\right) - 2\log_3 x = 2$

$$\frac{1}{2}\log_3 x \left(\log_3 x + 1\right) - 2\log_3 x = 2 \qquad\qquad t = \log_3 x$$

$$\frac{1}{2} t\,(t+1) - 2t = 2$$

$$t(t+1) - 4t - 4 = 0$$

$$t^2 + t - 4t - 4 = 0 \qquad t^2 - 3t - 4 = 0$$

$$(t-4)(t+1) = 0 \quad \begin{cases} t = 4 \\ t = -1 \end{cases}$$

$$\log_3 x = 4 \implies x = 3^4 = 81$$

$$\log_3 x = -1 \implies x = 3^{-1} = \frac{1}{3} \qquad \boxed{x = 81 \quad \lor \quad x = \frac{1}{3}}$$

380 $\dfrac{1}{5}\log_5(x+1) - \log_{x+1} 5 = \dfrac{4}{5}$ $\left[-\dfrac{4}{5}; 3124\right]$

C.E.

$\begin{cases} x+1>0 \\ x+1 \neq 1 \end{cases}$ $\begin{cases} x>-1 \\ x \neq 0 \end{cases}$

$\dfrac{1}{5}\log_5(x+1) - \dfrac{\log_5 5}{\log_5(x+1)} = \dfrac{4}{5}$

$\dfrac{1}{5}\log_5(x+1) - \dfrac{1}{\log_5(x+1)} = \dfrac{4}{5}$

$t = \log_5(x+1)$

$\dfrac{1}{5}t - \dfrac{1}{t} = \dfrac{4}{5}$

$\dfrac{t^2-5}{5t} = \dfrac{4t}{5t}$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5$ $\log_5(x+1) = 5 \implies x+1 = 5^5$

$\Downarrow$

$x = 3125 - 1 = 3124$

$t = -1$ $\log_5(x+1) = -1$

$\Downarrow$

$x + 1 = 5^{-1}$

$x = \dfrac{1}{5} - 1 = -\dfrac{4}{5}$

$\boxed{x = -\dfrac{4}{5} \quad \vee \quad x = 3124}$