398 $\frac{3}{\ln x} + \frac{\ln x}{\ln x + 1} = 2 + \frac{1}{\ln x}$ $[e^{\sqrt{2}}; e^{-\sqrt{2}}]$

C.E.

$$\begin{cases} x > 0 \\ \ln x \neq 0 \\ \ln x \neq -1 \end{cases}$$

$$\begin{cases} x > 0 \\ x \neq 1 \\ x \neq e^{-1} = \frac{1}{e} \end{cases}$$

$\ln x = t$

$$\frac{3}{t} + \frac{t}{t+1} = 2 + \frac{1}{t}$$

$$\frac{3(t+1) + t^2}{t\cancel{(t+1)}} = \frac{2t(t+1) + t + 1}{t\cancel{(t+1)}}$$

$$\cancel{3t} + 3 + t^2 = 2t^2 + \cancel{2t} + \cancel{t} + 1$$

$$t^2 = 2 \qquad t = \pm\sqrt{2}$$

$$\ln x = -\sqrt{2} \quad \vee \quad \ln x = \sqrt{2}$$

$$\boxed{x = e^{-\sqrt{2}} \quad \vee \quad x = e^{\sqrt{2}}}$$

397 $\dfrac{\log_2 x}{\log_2 x + 3} + \dfrac{6}{\log_2 x - 3} + \dfrac{72}{9 - \log_2^2 x} = 0$

$\left[\dfrac{1}{512}; 64\right]$

$$\frac{\log_2 x}{\log_2 x + 3} + \frac{6}{\log_2 x - 3} - \frac{72}{\log_2^2 x - 9} = 0$$

C.E.

$$\begin{cases} x > 0 \\ \log_2 x \neq -3 \\ \log_2 x \neq 3 \end{cases} \quad \begin{cases} x > 0 \\ x \neq 2^{-3} \\ x \neq 2^3 \end{cases} \quad \begin{cases} x > 0 \\ x \neq \frac{1}{8} \\ x \neq 8 \end{cases}$$

$t = \log_2 x$

$$\frac{t}{t+3} + \frac{6}{t-3} - \frac{72}{(t-3)(t+3)} = 0$$

$$\frac{t(t-3) + 6(t+3) - 72}{\cancel{(t+3)(t-3)}} = 0$$

$$t^2 - 3t + 6t + 18 - 72 = 0$$

$$t^2 + 3t - 54 = 0 \qquad \Delta = 9 + 216 = 225 = 15^2$$

$$t = \frac{-3 \pm 15}{2} = \begin{cases} -\frac{18}{2} = -9 \\ \frac{12}{2} = 6 \end{cases}$$

$$\log_2 x = -9 \quad \vee \quad \log_2 x = 6$$

$$x = 2^{-9} \quad \vee \quad x = 2^6$$

$$\boxed{x = \frac{1}{2^9} = \frac{1}{512} \quad \vee \quad x = 64}$$

461 $\log_3(2x-3) - \log_3(x+1) < 2$ $\left[x > \frac{3}{2}\right]$

$$\log_3(2x-3) - \log_3(x+1) < 2$$

C.E.

$$\begin{cases} 2x-3>0 \\ x+1>0 \end{cases} \quad \begin{cases} x>\frac{3}{2} \\ x>-1 \end{cases} \Rightarrow \boxed{x>\frac{3}{2}}$$

$$\log_3 \frac{2x-3}{x+1} < 2 \qquad \boxed{2 = 2\cdot\overbrace{\log_3 3}^{1} = \log_3 3^2}$$

$$\log_3 \frac{2x-3}{x+1} < \log_3 3^2$$

BASE $3>1$

$$\frac{2x-3}{x+1} < 3^2$$

$$\begin{cases} \frac{2x-3}{x+1} < 9 \\ x>\frac{3}{2} \end{cases} \quad \begin{cases} 2x-3<9(x+1) \\ x>\frac{3}{2} \end{cases} \quad \begin{cases} 2x-3<9x+9 \\ x>\frac{3}{2} \end{cases}$$

C.E.

$$\begin{cases} -7x<12 \\ x>\frac{3}{2} \end{cases} \quad \begin{cases} x>-\frac{12}{7} \\ x>\frac{3}{2} \end{cases} \Rightarrow \boxed{x>\frac{3}{2}}$$

465 $\log_{\frac{1}{4}}(x+1) - 2\log_{\frac{1}{4}}(x-2) + \log_{\frac{1}{4}}(x-1) < 0$

C.E. $\begin{cases} x+1>0 \\ x-2>0 \\ x-1>0 \end{cases}$ $\begin{cases} x>-1 \\ x>2 \\ x>1 \end{cases} \Rightarrow x>2$

$$\log_{\frac{1}{4}}(x+1) - \log_{\frac{1}{4}}(x-2)^2 + \log_{\frac{1}{4}}(x-1) < 0$$

$$\log_{\frac{1}{4}} \frac{(x+1)(x-1)}{(x-2)^2} < \log_{\frac{1}{4}} 1$$

BASE $0 < \frac{1}{4} < 1$

INVERTO LA DIS.

$\begin{cases} \frac{(x+1)(x-1)}{(x-2)^2} > 1 \\ x>2 \end{cases}$ $\begin{cases} x^2-1 > (x-2)^2 \\ x>2 \end{cases}$

$\begin{cases} \cancel{x^2}-1 > \cancel{x^2}+4-4x \\ x>2 \end{cases}$ $\begin{cases} 4x>5 \\ x>2 \end{cases}$ $\begin{cases} x>\frac{5}{4} \\ x>2 \end{cases} \Rightarrow \boxed{x>2}$

426 $\begin{cases} \log_2 x - \log_2 y = 2 \\ x - 2y = 1 \end{cases}$ $\left[\left(2; \frac{1}{2}\right)\right]$

C.E. $\begin{cases} x > 0 \\ y > 0 \end{cases}$

$\begin{cases} \log_2 (1+2y) - \log_2 y = 2 \\ x = 1 + 2y \end{cases}$ $\begin{cases} \log_2 \dfrac{1+2y}{y} = \log_2 2^2 \\ x = 1+2y \end{cases}$

$\dfrac{1+2y}{y} = 4$ $1+2y = 4y \Rightarrow 2y = 1 \Rightarrow y = \frac{1}{2}$

$\begin{cases} y = \frac{1}{2} \\ x = 1 + 2 \cdot \frac{1}{2} = 2 \end{cases}$ $\boxed{\begin{cases} x = 2 \\ y = \frac{1}{2} \end{cases}}$