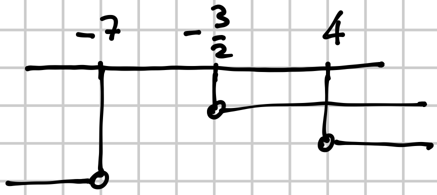
5 Risolvi le seguenti disequazioni:

...../20 **a.** $\log_3(2x+3) < \log_3(x-4)$; **b.** $\log_{\frac{1}{2}}(3x) - \log_{\frac{1}{2}}(x+2) > 1$.

a) $\begin{cases} 2x+3>0 \\ x-4>0 \\ 2x+3 < x-4 \end{cases}$ $\begin{cases} x > -\frac{3}{2} \\ x>4 \\ x<-7 \end{cases}$



IMPOSSIBILE

b) $\begin{cases} 3x>0 \\ x+2>0 \\ \log_{\frac{1}{2}} \dfrac{3x}{x+2} > 1 \end{cases}$ $\begin{cases} x>0 \\ x>-2 \\ \dfrac{3x}{x+2} < \dfrac{1}{2} \end{cases}$ $\begin{cases} x>0 \\ 6x < x+2 \end{cases}$

perché $\frac{1}{2} < 1$

(si inverte)

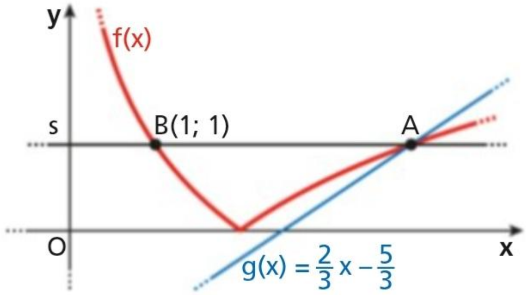
$\begin{cases} x>0 \\ 5x<2 \end{cases}$ $\begin{cases} x>0 \\ x<\frac{2}{5} \end{cases}$ $\Rightarrow$ $\boxed{0 < x < \frac{2}{5}}$

95 La funzione $f(x)$ in figura ha equazione $f(x) = |\log_a x + b|$, con $a > 0$ e $b < 0$.

a. Sapendo che la retta s è parallela all'asse x, ricava i valori dei parametri a e b.

b. Sia $h(x) = (f \circ g)(x)$. Scrivi l'espressione analitica di h e risolvi $h(x) > 1$.

[a) $a = 2, b = -1$; b) $\frac{5}{2} < x < 4 \vee x > \frac{17}{2}$]



a) $y = |\log_a x + b|$ $\quad B(1,1)$

SOSTITUENDO

$1 = |\overbrace{\log_a 1}^{0} + b|$

$1 = |b| \Rightarrow b = -1$ essendo $b < 0$

$A(x, 1)$ appartiene a $y = \frac{2}{3}x - \frac{5}{3} \Rightarrow 1 = \frac{2}{3}x - \frac{5}{3}$

$\frac{2}{3}x = 1 + \frac{5}{3} \qquad \frac{2}{3}x = \frac{8}{3}$

$x = 4$

$A(4, 1) \xrightarrow{\text{SOSTITUISCO}} y = |\log_a x - 1|$

$1 = |\log_a 4 - 1|$

$\Downarrow$

$\log_a 4 - 1 = \pm 1$

$\log_a 4 \cancel{-1} = \cancel{-1}$ $\qquad$ $\log_a 4 - 1 = 1$

$\log_a 4 = 0$ $\qquad$ $\log_a 4 = 2$

$\Downarrow$ $\qquad$ $\Downarrow$

$a^0 = 4$ IMPOSSIBILE $\qquad$ $a^2 = 4 \Rightarrow a = 2$

La funzione è $f : (0, +\infty) \to \mathbb{R}$ $\quad f(x) = |\log_2 x - 1|$

$f:(0,+\infty)\to\mathbb{R}$ $\quad f(x)=|\log_2 x-1|$

$g:\mathbb{R}\to\mathbb{R}$ $\quad g(x)=\frac{2}{3}x-\frac{5}{3}$ $\quad g(x)>0 \Longleftrightarrow \frac{2}{3}x-\frac{5}{3}>0$

$\Updownarrow$

$x>\frac{5}{2}$

DOMINIO DI h

$h(x)=(f\circ g)(x)=f(g(x))=\left|\log_2\left(\frac{2}{3}x-\frac{5}{3}\right)-1\right|$

$h:\left(\frac{5}{2},+\infty\right)\to\mathbb{R}$

$h(x)>1$ $\quad \left|\log_2\left(\frac{2}{3}x-\frac{5}{3}\right)-1\right|>1$

$\log_2\left(\frac{2}{3}x-\frac{5}{3}\right)-1>1 \quad \vee \quad \log_2\left(\frac{2}{3}x-\frac{5}{3}\right)-1<-1$

$\log_2\left(\frac{2}{3}x-\frac{5}{3}\right)>2 \qquad \log_2\left(\frac{2}{3}x-\frac{5}{3}\right)<0$

$$\begin{cases}\frac{2}{3}x-\frac{5}{3}>4\\ x>\frac{5}{2}\end{cases} \quad \vee \quad \begin{cases}\frac{2}{3}x-\frac{5}{3}<1\\ x>\frac{5}{2}\end{cases}$$

$$\begin{cases}\frac{2}{3}x>4+\frac{5}{3}\\ x>\frac{5}{2}\end{cases} \qquad \begin{cases}\frac{2}{3}x<1+\frac{5}{3}\\ x>\frac{5}{2}\end{cases}$$

$$\begin{cases}\frac{2}{3}x>\frac{17}{3}\\ x>\frac{5}{2}\end{cases} \qquad \begin{cases}\frac{2}{3}x<\frac{8}{3}\\ x>\frac{5}{2}\end{cases}$$

$$\begin{cases}x>\frac{17}{2}\\ x>\frac{5}{2}\end{cases} \quad x>\frac{17}{2} \qquad \begin{cases}x<4\\ x>\frac{5}{2}\end{cases} \quad \frac{5}{2}<x<4$$

$$x>\frac{17}{2} \quad \vee \quad \frac{5}{2}<x<4$$

$$\boxed{\frac{5}{2}<x<4 \quad \vee \quad x>\frac{17}{2}}$$