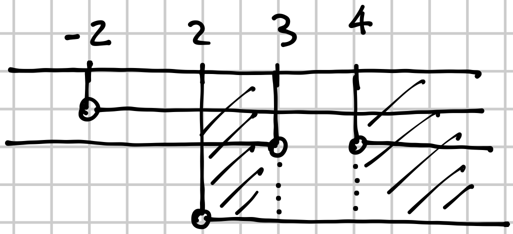
39 $\log_9(x+2)-\log_9(x^2-7x+12) \le \log_9 \dfrac{1}{x-2}+\dfrac{1}{2}$

$\left[2 < x \le \dfrac{5}{2} \vee x \ge 8\right]$

C.E.
$$\begin{cases} x+2>0 \\ x^2-7x+12>0 \\ x-2>0 \end{cases} \qquad \begin{cases} x>-2 \\ (x-3)(x-4)>0 \\ x>2 \end{cases} \qquad \begin{cases} x>-2 \\ x<3 \vee x>4 \\ x>2 \end{cases}$$



C.E. $2<x<3 \vee x>4$

$$\log_9 \frac{x+2}{x^2-7x+12} \le \log_9 \frac{1}{x-2} + \underbrace{\log_9 9^{\frac{1}{2}}}_{\log_9 3}$$

$$\log_9 \frac{x+2}{x^2-7x+12} \le \log_9 \frac{3}{x-2}$$

$$\begin{cases} \dfrac{x+2}{x^2-7x+12} \le \dfrac{3}{x-2} \\ 2<x<3 \vee x>4 \end{cases} \qquad \begin{cases} (x-2)(x+2) \le 3(x^2-7x+12) \\ 2<x<3 \vee x>4 \end{cases}$$

$$\begin{cases} x^2-4 \le 3x^2-21x+36 \\ 2<x<3 \vee x>4 \end{cases} \qquad \begin{cases} 2x^2-21x+40 \ge 0 \\ 2<x<3 \vee x>4 \end{cases} \qquad \begin{cases} x \le \frac{5}{2} \vee x \ge 8 \\ 2<x<3 \vee x>4 \end{cases}$$

$\Delta = 441-320 = 121 = 11^2 \qquad x = \dfrac{21 \pm 11}{4} = \begin{cases} \frac{5}{2} \\ 8 \end{cases} \qquad 2 < x \le \dfrac{5}{2} \vee x \ge 8$

25 $2\log_4^2|x+1| + \log_4|x^2-1| + \log_{\frac{1}{4}}|x-1| - 1 = 0$ $\left[-\frac{3}{4}; -\frac{5}{4}; -3\right]$

C.E. $x \neq \pm 1$

$$2\log_4^2|x+1| + \log_4|x-1||x+1| + \frac{\log_4|x-1|}{\log_4\frac{1}{4}} - 1 = 0$$

$$2\log_4^2|x+1| + \cancel{\log_4|x-1|} + \log_4|x+1| - \cancel{\log_4|x-1|} - 1 = 0$$

$$2\log_4^2|x+1| + \log_4|x+1| - 1 = 0 \qquad t = \log_4|x+1|$$

$$2t^2 + t - 1 = 0$$

$$2t^2 + 2t - t - 1 = 0 \qquad 2t(t+1) - (t+1) = 0 \qquad (t+1)(2t-1) = 0$$

$$t = -1 \quad \vee \quad t = \frac{1}{2}$$

$$\log_4|x+1| = -1 \quad \vee \quad \log_4|x+1| = \frac{1}{2}$$

$$|x+1| = \frac{1}{4} \quad \vee \quad |x+1| = 4^{\frac{1}{2}}$$

$$x+1 = \pm\frac{1}{4} \quad \vee \quad x+1 = \pm 2$$

$$x+1 = -\frac{1}{4} \quad \vee \quad x+1 = \frac{1}{4} \quad \vee \quad x+1 = -2 \quad \vee \quad x+1 = 2$$

$$\boxed{x = -\frac{5}{4} \quad \vee \quad x = -\frac{3}{4} \quad \vee \quad x = -3} \quad \vee \quad \underbrace{x = 1}_{\text{NON ACC. PER C.E.}}$$

2 Verifica l'identità $\dfrac{\log_a x}{1+\log_a b} = \dfrac{\log_b x}{1+\log_b a}$.

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$$\log_a x\,(1+\log_b a) = \log_b x\,(1+\log_a b)$$

$$\log_a x + \log_a x \cdot \log_b a = \log_b x + \log_b x \cdot \log_a b$$

$$\log_a x + \log_a x \cdot \frac{\log_a a}{\log_a b} = \log_b x + \log_b x \cdot \frac{\log_b b}{\log_b a}$$

$$\log_a x + \frac{\log_a x}{\log_a b} = \log_b x + \frac{\log_b x}{\log_b a}$$

$$\log_a x + \log_b x = \log_b x + \log_a x \quad \text{VERA!}$$

Quindi anche la prima è vera, perché i passaggi possono essere invertiti