514 $\ln x + \dfrac{2}{\ln x} - 3 \leq 0$ $[0 < x < 1 \vee e \leq x \leq e^2]$

C.E. $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

$t = \ln x$

$$t + \frac{2}{t} - 3 \leq 0 \qquad \frac{t^2 + 2 - 3t}{t} \leq 0$$

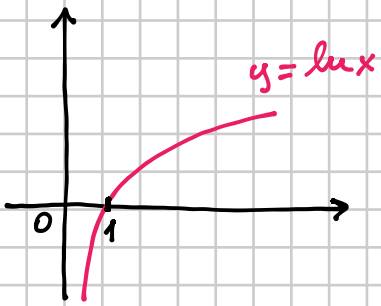
$$\frac{t^2 - 3t + 2}{t} \leq 0 \qquad \frac{(t-2)(t-1)}{t} \leq 0$$

(N_1, N_2, D)

		0	1	2
$t-2>0$	$t>2$	−	−	− 0 +
$t-1>0$	$t>1$	−	− 0	+ +
$t>0$	$t>0$	− ∄	+	+ +
		⊖ ∄	+ 0 ⊖	0 +

$t < 0 \quad \vee \quad 1 \leq t \leq 2$

$\ln x < 0 \quad \vee \quad 1 \leq \ln x \leq 2$



$0 < x < 1 \quad \vee \quad e \leq x \leq e^2$

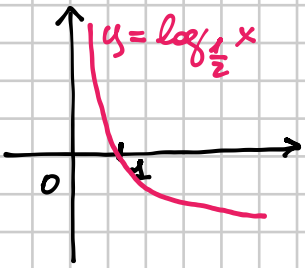
475 $\left(\log_{\frac{1}{2}} x\right)^2 - \log_{\frac{1}{2}} x - 2 < 0$ $\left[\frac{1}{4} < x < 2\right]$

$x > 0$ $\quad t = \log_{\frac{1}{2}} x$

$t^2 - t - 2 < 0$

$(t-2)(t+1) < 0$

$-1 < t < 2$ $\quad -1 < \log_{\frac{1}{2}} x < 2$

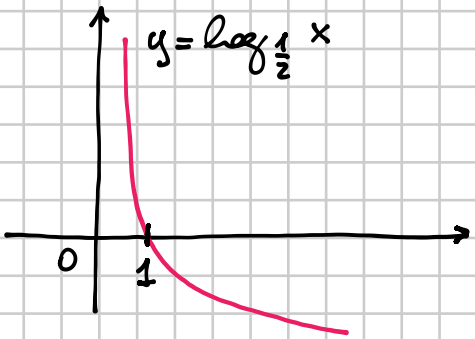


$\left(\frac{1}{2}\right)^{-1} > x > \left(\frac{1}{2}\right)^2$

$\boxed{\frac{1}{4} < x < 2}$

448 $\log_{\frac{1}{2}}(2x) > 0$ $\left[0 < x < \frac{1}{2}\right]$

$x > 0$



$\log_{\frac{1}{2}}(2x) > 0$

$\Downarrow$

$0 < 2x < 1$

$\boxed{0 < x < \frac{1}{2}}$

472 $(\log_2 x)^2 - \log_2 x < 0$ $[1 < x < 2]$

$x > 0$ $\quad t = \log_2 x$

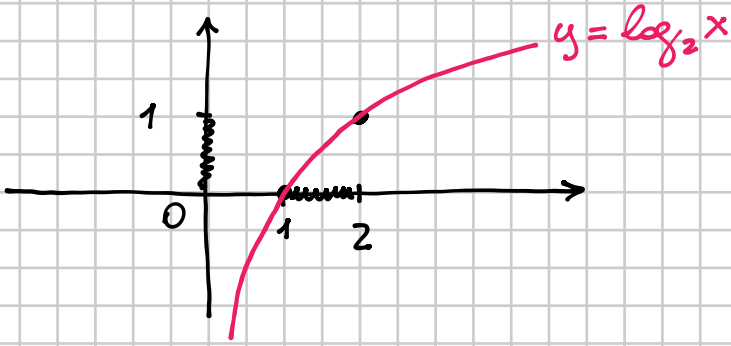
$t^2 - t < 0 \qquad t(t-1) < 0$

$0 < t < 1$

$0 < \log_2 x < 1$ ↓ applico l'esponenziale in base 2 (non inverto la disug.)

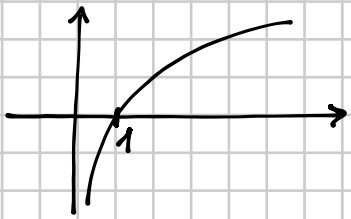
$2^0 < x < 2^1$

$\boxed{1 < x < 2}$



512 $\log\big(\log(x-1)\big) \geq 0$ $[x \geq 11]$

C.E. $\begin{cases} x-1>0 \\ \log(x-1)>0 \end{cases}$ $\begin{cases} x>1 \\ x-1>1 \end{cases}$ $\begin{cases} x>1 \\ x>2 \end{cases}$ $\Rightarrow x>2$



$\log\big(\log(x-1)\big) \geq 0$ ↓ applico $\exp_{10}$

$\log(x-1) \geq 10^0$

$\log(x-1) \geq 1$ ↓ applico $\exp_{10}$

$\begin{cases} x-1 \geq 10 \\ x>2 \end{cases}$ $\Rightarrow \boxed{x \geq 11}$

C.E.

515 $3(\log_3 x + \log_x 3) \geq 10$ $[1 < x \leq \sqrt[3]{3} \vee x \geq 27]$

C.E. $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

$$3\left(\log_3 x + \log_x 3\right) \geq 10$$

$$3\left(\log_3 x + \frac{1}{\log_3 x}\right) \geq 10$$

$$3\left(\log_3 x + \frac{1}{\log_3 x}\right) \geq 10$$

$t = \log_3 x$

$$3\left(t + \frac{1}{t}\right) \geq 10 \qquad 3t + \frac{3}{t} - 10 \geq 0$$

$$\frac{3t^2 - 10t + 3}{t} \geq 0$$

N

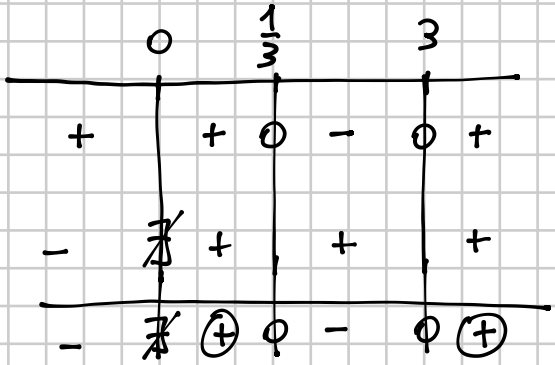
D

$N > 0 \quad 3t^2 - 10t + 3 > 0 \qquad \frac{\Delta}{4} = 25 - 9 = 16 \qquad t = \frac{5 \pm 4}{3} = \begin{cases} \frac{1}{3} \\ 3 \end{cases}$

$t < \frac{1}{3} \vee t > 3$

$N > 0 \quad t < \frac{1}{3} \vee t > 3$

$D > 0 \quad t > 0$



$0 < t \leq \frac{1}{3} \vee t \geq 3$

$0 < \log_3 x \leq \frac{1}{3} \vee \log_3 x \geq 3$

$$\boxed{1 < x \leq \sqrt[3]{3} \quad \vee \quad x \geq 27}$$

524 $\log^4 x - 8\log^2 x + 16 > 0$

$\left[x > 0 \wedge x \neq \frac{1}{100} \wedge x \neq 100\right]$

$x > 0$ $\qquad t = \log x$

$t^4 - 8t^2 + 16 > 0$

$(t^2 - 4)^2 > 0 \iff t^2 - 4 \neq 0 \iff t \neq \pm 2$

$\log x \neq \pm 2$

$\log x \neq -2 \wedge \log x \neq 2$

$x > 0 \wedge x \neq 10^{-2} \wedge x \neq 10^2$

$\boxed{x > 0 \wedge x \neq \frac{1}{100} \wedge x \neq 100}$

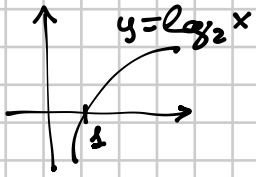
525 $\log_2 \sqrt{2x - x^2} < 0$ $[0 < x < 2 \wedge x \neq 1]$

C.E. $\begin{cases} 2x - x^2 \geq 0 \\ \sqrt{2x - x^2} > 0 \end{cases} \Rightarrow 2x - x^2 > 0$ $\quad x^2 - 2x < 0$

$x(x-2) < 0$

$0 < x < 2$ C.E.

$\log_2 \sqrt{2x - x^2} < 0$



$\Downarrow$

$0 < \sqrt{2x - x^2} < 1$

$\begin{cases} 2x - x^2 < 1 \\ 0 < x < 2 \end{cases}$ $\begin{cases} x^2 - 2x + 1 > 0 \\ 0 < x < 2 \end{cases}$ $\begin{cases} (x-1)^2 > 0 \\ 0 < x < 2 \end{cases}$ $\begin{cases} x \neq 1 \\ 0 < x < 2 \end{cases}$

$0 < x < 1 \quad \vee \quad 1 < x < 2$

$\boxed{0 < x < 2 \wedge x \neq 1}$