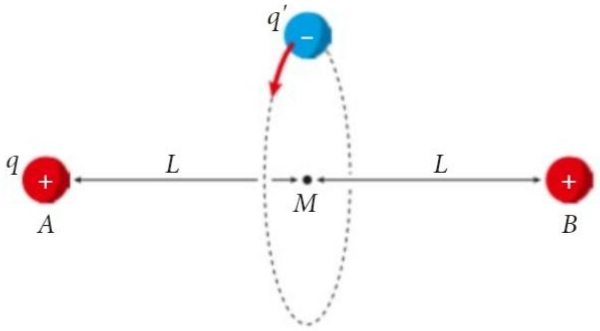
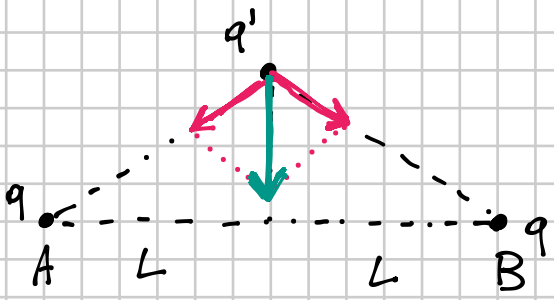
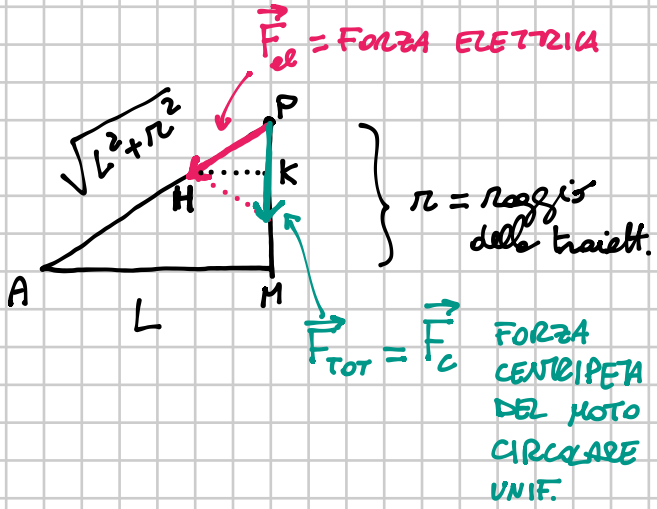
82 ◆◆◆ Due cariche identiche $q = 5{,}0 \times 10^{-6}$ C si trovano, nel vuoto, in due punti A e B, a distanza $2l = 12$ cm. Una sferetta di massa $m = 9{,}0$ mg e di carica negativa $q' = -4{,}0 \times 10^{-6}$ C, compie un moto circolare uniforme, attorno al segmento AB, in un piano perpendicolare ad AB e passante per il suo punto medio M. La frequenza del moto è $f = 1{,}0$ kHz. Trascura la forza-peso.



- Calcola la forza totale esercitata dalle cariche positive sulla carica negativa.
- Calcola il modulo della velocità della sferetta.

[28 N; $5{,}0 \times 10^2$ m/s]





3 triangoli AMP e HKP sono simili, dunque:

$$\overline{PK} : \overline{PM} = \overline{PH} : \overline{PA} \Rightarrow \frac{F_c}{2} : r = F_{el} : \sqrt{L^2+r^2} \quad (*)$$

$$F_c = m\frac{v^2}{r} \qquad v = \frac{2\pi r}{T} = 2\pi r f \Rightarrow F_c = m\frac{4\pi^2 r^2 f^2}{r} = 4\pi^2 m f^2 r$$

$$F_{el} = k_0 \frac{|q||q'|}{L^2+r^2}$$

$$(*)\ \boxed{\frac{F_c}{2} = \frac{r \cdot F_{el}}{\sqrt{L^2+r^2}}} \Rightarrow 2\pi^2 m f^2 \cancel{r} = \frac{\cancel{r} \cdot F_{el}}{\sqrt{L^2+r^2}}$$

$$\Rightarrow 2\pi^2 m f^2 = \frac{k_0|q||q'|}{(L^2+r^2)\sqrt{L^2+r^2}}$$ equazione nell'incognita r

Poniamo $L^2+r^2 = x$

$$2\pi^2 m f^2 = \frac{k_0|q||q'|}{x\sqrt{x}} \Rightarrow x\sqrt{x} = \frac{k_0|q||q'|}{2\pi^2 m f^2}$$

$$x\sqrt{x} = \frac{K_0|q||q'|}{2\pi^2 m f^2}$$

$$\sqrt{x^3} = \frac{K_0|q||q'|}{2\pi^2 m f^2} \Rightarrow x^3 = \left(\frac{K_0|q||q'|}{2\pi^2 m f^2}\right)^2$$

$$x = \sqrt[3]{\left(\frac{K_0|q||q'|}{2\pi^2 m f^2}\right)^2} = \sqrt[3]{\left(\frac{(8{,}99\times10^9)\cdot(20\times10^{-12})}{2\pi^2 (9{,}0\times10^{-6})(1{,}0\times10^3)^2}\right)^2}\ m^2 =$$

$$= 0{,}010080412\ m^2 \qquad r = \sqrt{x - L^2} = \sqrt{0{,}010080412 - (0{,}060)^2}\ m =$$

$$= 0{,}08050100 7\ m$$

$$F_c = 4\pi^2 m r f^2 = 4\pi^2 (9{,}0\times10^{-6}\ kg)(0{,}08050100 7\ m)(1{,}0\times10^3\ Hz)^2 =$$

$$= 28{,}6024\ldots\ N \simeq \boxed{29\,N}$$

$$v = 2\pi f r = 2\pi\,(1{,}0\times10^3\ Hz)(0{,}08050100 7\ m) =$$

$$= 0{,}5058\ldots \times10^3\ \frac{m}{s} \simeq \boxed{5{,}1\times10^2\ \frac{m}{s}}$$