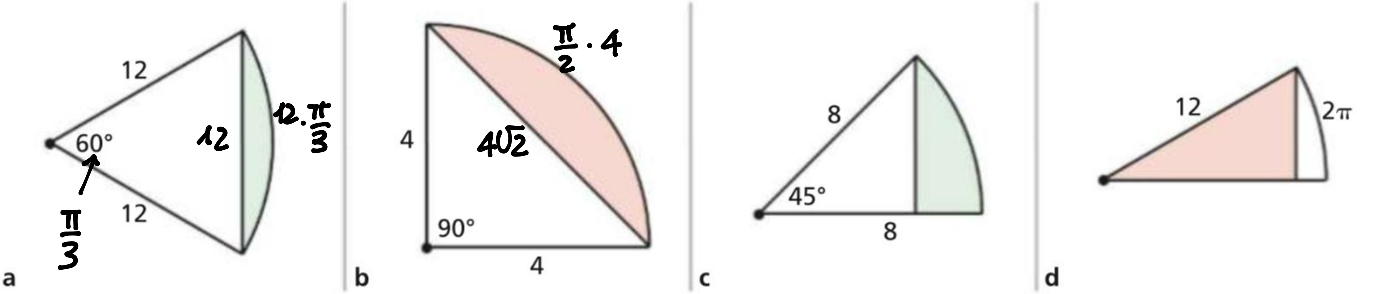
61 Trova il perimetro e l'area delle zone colorate.



[a) $12 + 4\pi$, $24\pi - 36\sqrt{3}$; b) $2\pi + 4\sqrt{2}$, $4\pi - 8$; c) $2\pi + 8$, $8\pi - 16$; d) $18 + 6\sqrt{3}$, $18\sqrt{3}$]

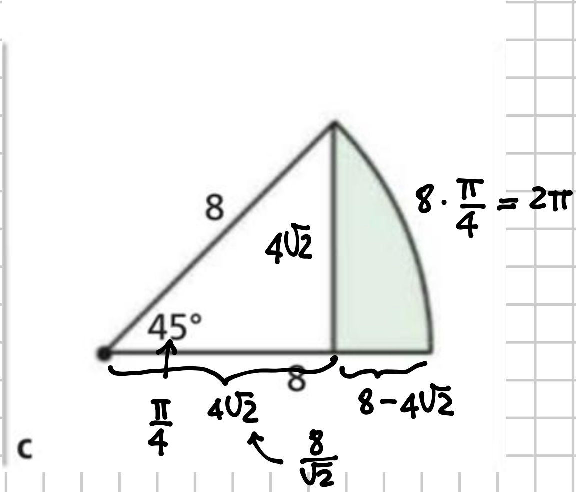
$\text{arco} = l = r\alpha$ ↑ ANGOLO IN RADIANTI

$\mathcal{A}_{\text{REA SETTORE CIRCOLARE}} = \frac{1}{2} r^2 \alpha$ ↑ ANGOLO IN RADIANTI

a) $2p = 12 + 4\pi$; $\mathcal{A} = \frac{1}{2}\cdot 12^2 \cdot \frac{\pi}{3} - \frac{1}{2}\cdot 12 \cdot 12 \frac{\sqrt{3}}{2} = 24\pi - 36\sqrt{3}$

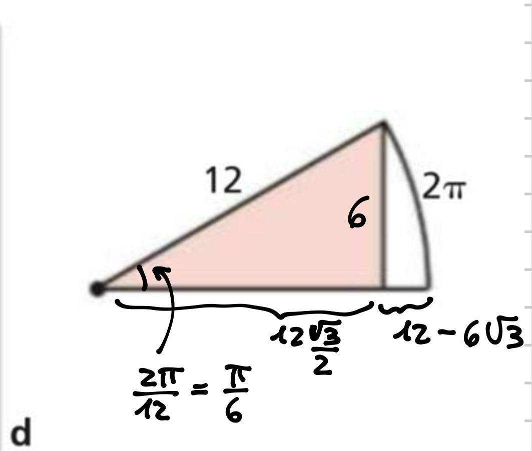
b) $2p = 2\pi + 4\sqrt{2}$; $\mathcal{A} = \frac{4^2\pi}{4} - \frac{4\cdot 4}{2} = 4\pi - 8$

c)



$2p = 4\sqrt{2} + 8 - 4\sqrt{2} + 2\pi = 8 + 2\pi$

$\mathcal{A} = \frac{1}{2} 8^2 \cdot \frac{\pi}{4} - \frac{(4\sqrt{2})^2}{2} = 8\pi - 16$

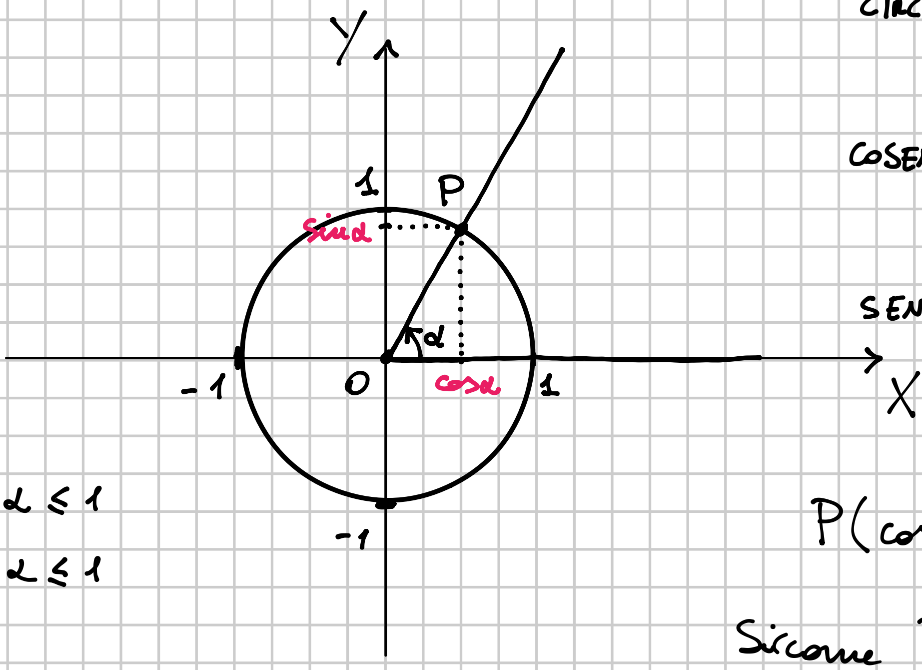


$2p = 12 + 6 + 6\sqrt{3} = 18 + 6\sqrt{3} = 6(3+\sqrt{3})$

$\mathcal{A} = \frac{1}{2}\cdot 6 \cdot 6\sqrt{3} = 18\sqrt{3}$

SENO E COSENO

CIRC. GONIOMETRICA $X^2+Y^2=1$



COSENO DI α ($\cos\alpha$) = ASCISSA DI P

SENO DI α ($\sin\alpha$) = ORDINATA DI P

$-1 \leq \sin\alpha \leq 1$

$-1 \leq \cos\alpha \leq 1$

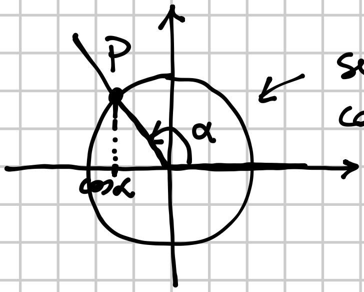
$P(\cos\alpha, \sin\alpha)$

Siccome $P \in$ CIRC. GONIOMETRICA deve soddisfare l'equazione

$$X^2+Y^2=1$$

$$\boxed{\cos^2\alpha + \sin^2\alpha = 1}$$

1° RELAZIONE FONDAMENTALE DELLA TRIGONOMETRIA



$\sin\alpha > 0$
$\cos\alpha < 0$

Se considero "più giri"...

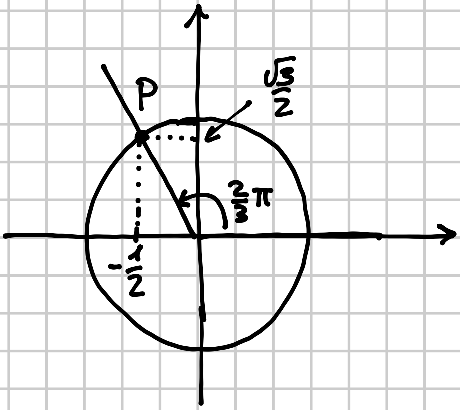
$\sin(\alpha+2\pi) = \sin\alpha \Rightarrow \sin(\alpha+2k\pi) = \sin\alpha$

$\cos(\alpha+2\pi) = \cos\alpha \Rightarrow \cos(\alpha+2k\pi) = \cos\alpha$

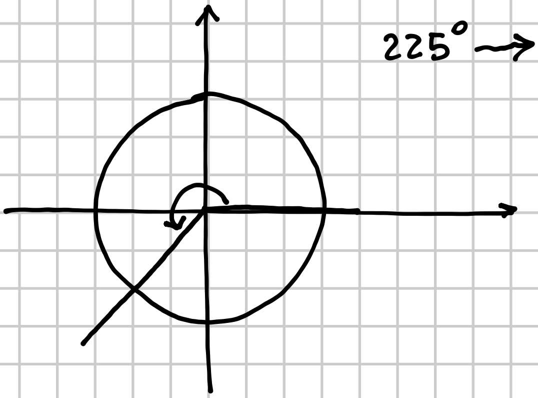
$k \in \mathbb{Z}$

ANGOLO (GRADI)	ANGOLO (RAD.)	COSENO	SENO
$0°$	0	1	0
$90°$	$\frac{\pi}{2}$	0	1
$180°$	π	-1	0
$270°$	$\frac{3}{2}\pi$	0	-1
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$\sin 120° = ?$ $\cos 120° = ?$ $120° \rightsquigarrow \frac{2}{3}\pi$ rad.



$\sin \frac{2}{3}\pi = \frac{\sqrt{3}}{2}$ $\cos \frac{2}{3}\pi = -\frac{1}{2}$



$225° \rightsquigarrow \frac{5}{4}\pi$

$\sin \frac{5}{4}\pi = -\frac{\sqrt{2}}{2}$

$\cos \frac{5}{4}\pi = -\frac{\sqrt{2}}{2}$

Utilizza i dati nelle figure per determinare i valori richiesti.

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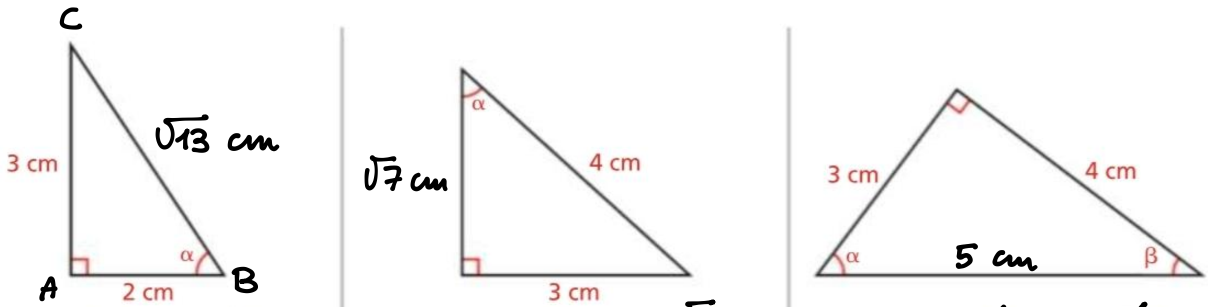

a $\sin\alpha = \frac{3}{\sqrt{13}}$ $\cos\alpha = \frac{2}{\sqrt{13}}$

b $\sin\alpha = \frac{3}{4}$ $\cos\alpha = \frac{\sqrt{7}}{4}$

c $\sin\alpha = \frac{4}{5}$ $\cos\beta = \frac{4}{5}$

$\overline{AC} = \overline{BC} \cdot \sin\alpha$

$\sin\alpha = \frac{\overline{AC}}{\overline{BC}}$

121 $\frac{4}{3}\cos(-90°)+\sin(-270°)-\frac{3}{4}\sin(-450°)+\frac{1}{4}\sin 270° =$ $\left[\frac{3}{2}\right]$

$$= \frac{4}{3}\cdot 0 + 1 - \frac{3}{4}\cdot(-1) + \frac{1}{4}(-1) = 1 + \frac{3}{4} - \frac{1}{4} = 1 + \frac{1}{2} = \frac{3}{2}$$

$-450° = -360° - 90°$ $\quad$ $\sin(-450°) = \sin(-90° - 360°) =$

$$= \sin(-90°) = -1$$

125 $\cos 4\pi + 2\sin\left(-\frac{15}{2}\pi\right) + \frac{1}{3}\cos(-3\pi) + \sin\frac{9}{2}\pi =$ $\left[\frac{11}{3}\right]$

$$= 1 + 2\cdot 1 + \frac{1}{3}\cdot(-1) + 1 = 1 + 2 - \frac{1}{3} + 1 = 4 - \frac{1}{3} = \frac{11}{3}$$

$$-\frac{15}{2}\pi = -\left(7 + \frac{1}{2}\right)\pi = -\frac{\pi}{2} - 7\pi = -\frac{\pi}{2} - \pi - 6\pi = -\frac{3}{2}\pi - 6\pi$$

$$\sin\left(-\frac{15}{2}\pi\right) = \sin\left(-\frac{3}{2}\pi - 6\pi\right) = \sin\left(-\frac{3}{2}\pi\right) = 1$$

$$\frac{9}{2}\pi = \left(4 + \frac{1}{2}\right)\pi = 4\pi + \frac{\pi}{2} \rightsquigarrow \sin\frac{9}{2}\pi = \sin\frac{\pi}{2} = 1$$

127 $\left(a\cos 2\pi + b\sin\frac{7}{2}\pi\right)^2 - \left[a\sin\left(-\frac{3}{2}\pi\right) + b\cos(-5\pi)\right]^2$

$$(a\cdot 1 + b\cdot(-1))^2 - [a\cdot 1 + b\cdot(-1)]^2 = (a-b)^2 - (a-b)^2 = 0$$

$$\sin\frac{7}{2}\pi = \sin\left(\frac{\pi}{2} + 3\pi\right) = \sin\left(\frac{\pi}{2} + \pi + 2\pi\right) = \sin\left(\frac{3}{2}\pi + 2\pi\right) = -1$$

$$\cos(-5\pi) = \cos(-\pi - 4\pi) = \cos(-\pi) = -1$$