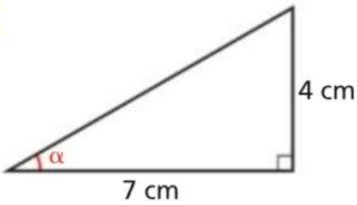
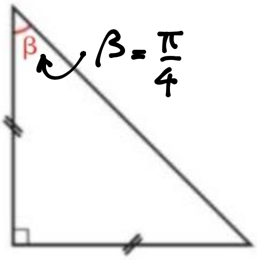
Nei seguenti triangoli calcola la tangente dell'angolo indicato.

174



$\tan\alpha = \frac{4}{7}$

175

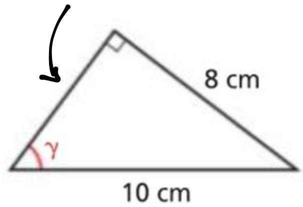


$\beta = \frac{\pi}{4}$

$\tan\beta = 1$

176

6 cm (TH. PITAGORA)



$\tan\gamma = \frac{8}{6} = \frac{4}{3}$

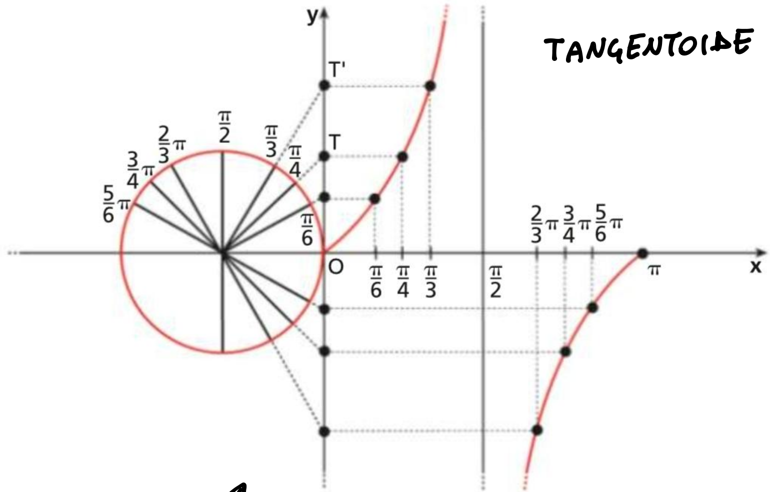
$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \alpha \neq \frac{\pi}{2} + k\pi$

α (GRADI)	α (RAD.)	$\tan\alpha$
0°	0	0
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	1
60°	$\frac{\pi}{3}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	$\nexists$
135°	$\frac{3}{4}\pi$	-1
180°	π	0
270°	$\frac{3}{2}\pi$	$\nexists$
360°	2π	0

$$\tan\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

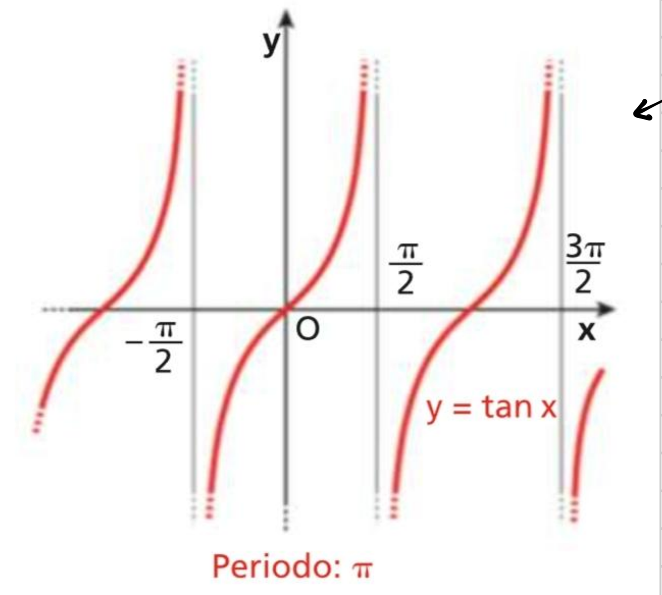
$$\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$y = \tan x \qquad D = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\}$



TANGENTOIDE

GRAFICO SU $[0, \pi] \setminus \{\frac{\pi}{2}\}$



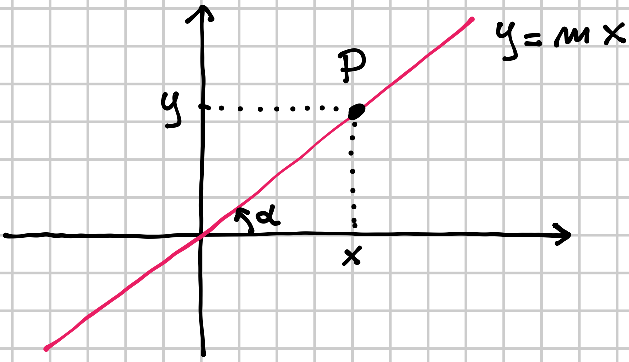
← GRAFICO SU $D = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\}$

$\tan x$ È PERIODICA DI PERIODO π

$\tan(x + k\pi) = \tan x \quad \forall x \in D$

$-\infty < \tan x < +\infty$

SIGNIFICATO GEOMETRICO DEL COEFF. ANGOLARE DI UNA RETTA

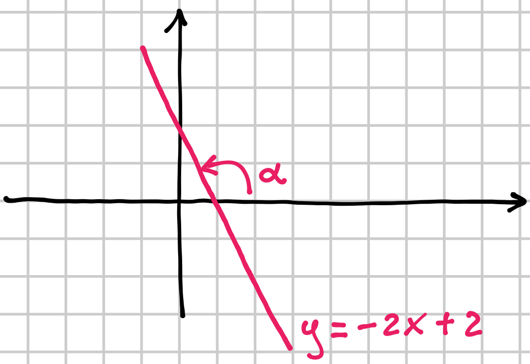


m = coeff. angolare

$m = \frac{y}{x}$

$\tan\alpha = \frac{y}{x} = m$

Il coeff. angolare di una retta è la tangente dell'angolo che la retta forma con la direzione positiva dell'asse x



$\tan\alpha = -2$

Trasforma le seguenti espressioni in funzione soltanto di $\sin\alpha$, sapendo che $0 < \alpha < \frac{\pi}{2}$:

206 $\dfrac{\tan\alpha + \cos\alpha}{\tan^2\alpha} \cdot \dfrac{1}{\cos\alpha} - \dfrac{1}{\tan^2\alpha} =$ $\left[\dfrac{1}{\sin\alpha}\right]$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha} + \cos\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}} \cdot \frac{1}{\cos\alpha} - \frac{1}{\frac{\sin^2\alpha}{\cos^2\alpha}} =$$

$$= \frac{\frac{\sin\alpha + \cos^2\alpha}{\cos\alpha}}{\frac{\sin^2\alpha}{\cancel{\cos^2\alpha}}} \cdot \frac{1}{\cancel{\cos\alpha}} - \frac{\cos^2\alpha}{\sin^2\alpha} =$$

$$= \frac{\sin\alpha + \cos^2\alpha}{\cancel{\cos\alpha}} \cdot \frac{\cancel{\cos\alpha}}{\sin^2\alpha} - \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{\sin\alpha + \cancel{\cos^2\alpha} - \cancel{\cos^2\alpha}}{\sin^2\alpha} = \frac{\cancel{\sin\alpha}}{\sin^{\cancel{2}}\alpha} =$$

$$= \frac{1}{\sin\alpha}$$

OSSERVAZIONE

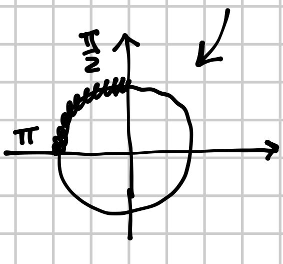
Dalla relazione $\sin^2\alpha + \cos^2\alpha = 1$ si ricava

$$\sin\alpha = \pm\sqrt{1 - \cos^2\alpha}$$

$$\cos\alpha = \pm\sqrt{1 - \sin^2\alpha}$$

Per decidere se + o − devo sapere in quale intervallo varia α.

Ad esempio, se $\frac{\pi}{2} < \alpha < \pi$



$$\cos\alpha = -\sqrt{1 - \sin^2\alpha}$$

e

$$\sin\alpha = +\sqrt{1 - \cos^2\alpha}$$

Semplificare

213 $\dfrac{\cos^2\alpha}{1-\cos^2\alpha} - \tan\alpha + \dfrac{1-\sin^2\alpha}{\cos^2\alpha} - \dfrac{1}{\sin^2\alpha} =$

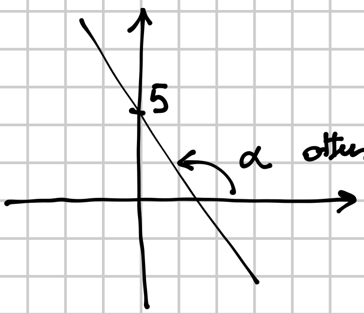
$$= \frac{\cos^2\alpha}{\sin^2\alpha} - \frac{\sin\alpha}{\cos\alpha} + \frac{\cancel{\cos^2\alpha}^{\,1}}{\cancel{\cos^2\alpha}_{\,1}} - \frac{1}{\sin^2\alpha} =$$

$$= \frac{\cos^2\alpha}{\sin^2\alpha} - \frac{\sin\alpha}{\cos\alpha} + 1 - \frac{1}{\sin^2\alpha} =$$

$$= \frac{\cos^2\alpha - 1}{\sin^2\alpha} - \frac{\sin\alpha}{\cos\alpha} + 1 = \frac{-\cancel{\sin^2\alpha}^{\,1}}{\cancel{\sin^2\alpha}_{\,1}} - \frac{\sin\alpha}{\cos\alpha} + 1 =$$

$$= \cancel{-1} - \frac{\sin\alpha}{\cos\alpha} \cancel{+1} = \boxed{-\tan\alpha}$$

224 Calcola il coseno dell'angolo che la retta di equazione $y = -\dfrac{3}{4}x + 5$ forma con l'asse x. $\left[-\dfrac{4}{5}\right]$



$\Rightarrow$ $\cos\alpha < 0$, $\sin\alpha > 0$

$\tan\alpha = -\dfrac{3}{4} \Rightarrow \begin{cases} \dfrac{\sin\alpha}{\cos\alpha} = -\dfrac{3}{4} \\ \sin^2\alpha + \cos^2\alpha = 1 \end{cases}$

$\begin{cases} \sin\alpha = -\dfrac{3}{4}\cos\alpha \\ \left(-\dfrac{3}{4}\cos\alpha\right)^2 + \cos^2\alpha = 1 \end{cases}$

$\dfrac{9}{16}\cos^2\alpha + \cos^2\alpha = 1$ $\quad$ $\dfrac{25}{16}\cos^2\alpha = 1$

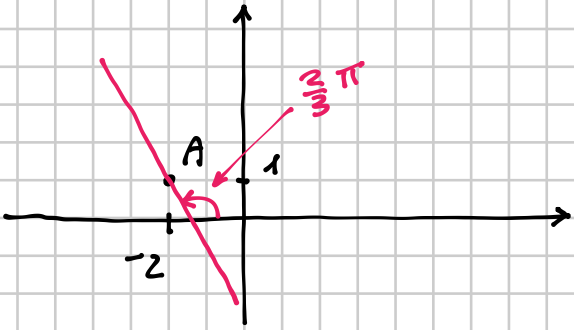
$\cos^2\alpha = \dfrac{16}{25}$

$\cos\alpha < 0$

$\cos\alpha = -\dfrac{4}{5}$

225 Trova l'equazione della retta passante per il punto $A(-2; 1)$ e che forma un angolo di $\underbrace{\frac{2}{3}\pi}_{120°}$ con l'asse x.

$[y = -\sqrt{3}x - 2\sqrt{3} + 1]$



$$y - y_A = m(x - x_A)$$

$$m = \tan\left(\frac{2}{3}\pi\right) = \frac{\sin\frac{2}{3}\pi}{\cos\frac{2}{3}\pi} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$y - 1 = -\sqrt{3}(x + 2)$$

$$\boxed{y = -\sqrt{3}x - 2\sqrt{3} + 1}$$

SECANTE

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \alpha \neq \frac{\pi}{2} + k\pi$$

COSECANTE

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \alpha \neq k\pi$$

COTANGENTE

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad \alpha \neq k\pi$$

$$\left(\text{se } \alpha \neq k\frac{\pi}{2} \quad \cot\alpha = \frac{1}{\tan\alpha}\right)$$