4. Calcolare $\sin[\arctan(-\sqrt{2})]$; $\tan\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right]$; $\sin\left[\arctan\left(-\frac{12}{5}\right)\right]$. $\left[-\frac{\sqrt{6}}{3}; -\frac{\sqrt{3}}{3}; -\frac{12}{13}\right]$

$$\sin^2\alpha = \frac{\tan^2\alpha}{1+\tan^2\alpha}$$

$$\sin(\arctan(-\sqrt{2})) = -\sqrt{\frac{(-\sqrt{2})^2}{1+(-\sqrt{2})^2}} = -\sqrt{\frac{2}{3}} = -\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$$

$$\tan\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right] = \tan\left(\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{3}$$

$$\tan\left(\pi-\frac{\pi}{6}\right) = \tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6}$$

$$\sin\left(\arctan\left(-\frac{12}{5}\right)\right) = -\sqrt{\frac{\left(-\frac{12}{5}\right)^2}{1+\left(-\frac{12}{5}\right)^2}} = -\sqrt{\frac{\frac{144}{\cancel{25}}}{\frac{169}{\cancel{25}}}} = -\frac{12}{13}$$

5. Verificare che, per $-1 \le x \le 1$, è $\cos(\arcsin x) = \sqrt{1-x^2}$.

Il dominio dell'arcoseno è $[-1, 1]$, quindi $-1 \le x \le 1$

L'arcoseno di un qualsiasi valore è un angolo compreso tra $-\frac{\pi}{2}$ e $\frac{\pi}{2}$, e il coseno di un tale angolo è positivo (o nullo).

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

prendo + per quanto detto prima

$$\alpha = \arcsin x \implies \cos(\arcsin x) = \pm\sqrt{1-\sin^2(\arcsin x)}$$

$$\implies \cos(\arcsin x) = \sqrt{1-x^2}$$

6. Verificare che, per $-1 \leq x \leq 1$, è $\sin(\arccos x) = \sqrt{1-x^2}$.

Stesso procedimento di prima $\quad \sin\alpha = \pm\sqrt{1-\cos^2\alpha}$

prendo + perché arccos x restituisce un angolo compreso tra 0 e π, e il seno di tale angolo è positivo (o nullo)

7. Calcolare

$$\cos(\arcsin\sqrt{1-x^2}). \qquad \left[|x|, \text{ per } x \in [-1,1]\right]$$

$\cos(\arcsin x) = \sqrt{1-x^2}$ DIMOSTRATA ALL'ES. 5

$\cos(\arcsin\sqrt{1-x^2}) = \sqrt{1-(1-x^2)} = \sqrt{1-1+x^2} = \sqrt{x^2} = |x|$

$-1 \leq x \leq 1$

8. Determinare

$$\cos(\arctan\sqrt{1+x^2}). \qquad \left[\frac{1}{\sqrt{2+x^2}}\right]$$

$x \in \mathbb{R}$

$$\cos^2\alpha = \frac{1}{1+\tan^2\alpha} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{1+\tan^2\alpha}}$$

$\cos(\arctan\sqrt{1+x^2})$ è positivo perché $\arctan\sqrt{1+x^2}$ è compreso tra 0 e $\frac{\pi}{2}$.

$$\cos(\arctan\sqrt{1+x^2}) = \frac{1}{\sqrt{1+(\sqrt{1+x^2})^2}} = \frac{1}{\sqrt{1+1+x^2}} = \frac{1}{\sqrt{2+x^2}}$$

Verifica le seguenti uguaglianze.

482 $\left(\sin\frac{3}{4}\pi+\cos\frac{7}{4}\pi\right)^2=-\cos 5\pi+\cot\frac{5}{4}\pi$

$$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)^2=-(-1)+1 \Rightarrow (\sqrt{2})^2=1+1 \Rightarrow 2=2$$

ok!

$$\cos\frac{7}{4}\pi=\cos\left(2\pi-\frac{\pi}{4}\right)=\cos\left(-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

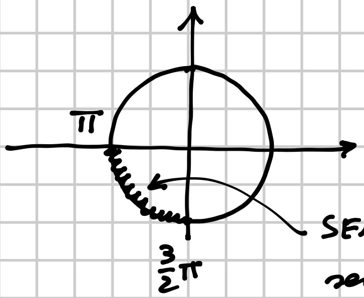
$$\cos 5\pi=\cos(4\pi+\pi)=\cos\pi=-1$$

$$\cot\frac{5}{4}\pi=\cot\left(\pi+\frac{\pi}{4}\right)=\cot\frac{\pi}{4}=1$$

CALCOLARE ↓ SAPENDO CHE ↓

10 $\sin\alpha$ e $\cos\alpha$; $\tan\alpha=\frac{28}{45}$, con $\pi<\alpha<\frac{3}{2}\pi$. $\left[-\frac{28}{53};-\frac{45}{53}\right]$

$$\begin{cases}\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{28}{45}\\[2ex] \sin^2\alpha+\cos^2\alpha=1\end{cases}$$



SENO E COSENO sono entrambi <0

$$\begin{cases}\sin\alpha=\dfrac{28}{45}\cos\alpha\\[2ex] \left(\dfrac{28}{45}\cos\alpha\right)^2+\cos^2\alpha=1\end{cases}$$

$$\frac{784}{2025}\cos^2\alpha+\cos^2\alpha=1$$

$$\frac{784+2025}{2025}\cos^2\alpha=1$$

$$\frac{2809}{2025}\cos^2\alpha=1$$

$$\cos^2\alpha=\frac{2025}{2809}$$

$$\cos\alpha=-\sqrt{\frac{2025}{2809}}=-\frac{45}{53}$$

$$\sin\alpha=\frac{28}{45}\cos\alpha=\frac{28}{45}\left(-\frac{45}{53}\right)=-\frac{28}{53}$$

CALCOLARE ↓ SAPENDO CHE ↓

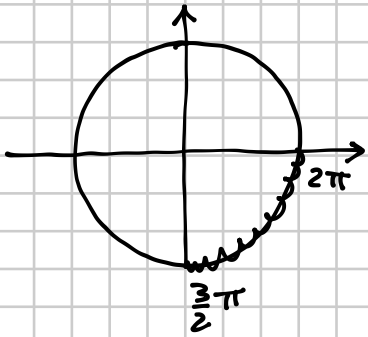
11 $\sin\alpha$ e $\tan\alpha$; $\cos\alpha = \frac{39}{89}$, con $\frac{3}{2}\pi < \alpha < 2\pi$. $\left[-\frac{80}{89}; -\frac{80}{39}\right]$

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} =$$

$$= -\sqrt{1-\left(\frac{39}{89}\right)^2} =$$

$$= -\sqrt{1-\frac{1521}{7921}} =$$

$$= -\sqrt{\frac{7921-1521}{7921}} = -\sqrt{\frac{6400}{7921}} = -\frac{80}{89}$$



$\sin\alpha < 0$

$\cos\alpha > 0$

$\tan\alpha < 0$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{80}{89}}{\frac{39}{89}} = -\frac{80}{39}$$

34 $$\cot^2\left(\frac{3}{2}\pi+\alpha\right) + \sec^2(\pi+\alpha) - \frac{\sin\left(\frac{\pi}{2}+\alpha\right)\cos(\pi-\alpha)}{\sin^2\left(\alpha-\frac{3}{2}\pi\right)} =$$

$$= \cot^2\left(\pi+\frac{\pi}{2}+\alpha\right) + \frac{1}{\cos^2(\pi+\alpha)} - \frac{\cos\alpha\cdot(-\cos\alpha)}{\left[-\sin\left(\frac{3}{2}\pi-\alpha\right)\right]^2} =$$

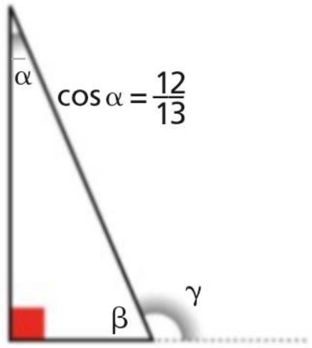
$$= \left[\cot\left(\frac{\pi}{2}+\alpha\right)\right]^2 + \frac{1}{[-\cos\alpha]^2} - \frac{-\cos^2\alpha}{\left[-\sin\left(\pi+\frac{\pi}{2}-\alpha\right)\right]^2} =$$

$$= [-\tan\alpha]^2 + \frac{1}{\cos^2\alpha} + \frac{\cos^2\alpha}{\left[-\left(-\sin\left(\frac{\pi}{2}-\alpha\right)\right)\right]^2} =$$

$$= \tan^2\alpha + \frac{1}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^2\alpha} + \frac{1}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} = \frac{2}{\cos^2\alpha} = 2\sec^2\alpha$$

1

45



Calcola:
$\tan\beta$, $\cos\gamma$, $\sin(\pi+\gamma)$.

$\left[\frac{12}{5}; -\frac{5}{13}; -\frac{12}{13}\right]$

$$\sin\beta = \sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha = \frac{12}{13}$$

$$\cos\beta = \sqrt{1-\sin^2\beta} =$$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} =$$

$$= \frac{5}{13}$$

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$$

$$\cos\gamma = \cos(\pi-\beta) = -\cos\beta = -\frac{5}{13}$$

$$\sin(\pi+\gamma) = -\sin\gamma = -\sqrt{1-\cos^2\gamma} = -\sqrt{1-\left(-\frac{5}{13}\right)^2} = -\sqrt{1-\frac{25}{169}} =$$

$$= -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

2 Calcola il valore delle seguenti espressioni.

......../20

a. $\sin\frac{\pi}{6} + \left(\cos\frac{\pi}{2} + \tan\frac{\pi}{3}\right)^2 - \cos\frac{2}{3}\pi \cdot \cot\frac{3}{4}\pi$

b. $\sin^2\frac{5}{3}\pi - \cot\frac{3}{2}\pi + \cos\frac{11}{6}\pi \cdot \tan\frac{\pi}{6}$

a) $\frac{1}{2} + (0+\sqrt{3})^2 - \left(-\frac{1}{2}\right)\cdot(-1) = \frac{1}{2} + 3 - \frac{1}{2} = 3$

b) $\left(-\frac{\sqrt{3}}{2}\right)^2 - 0 + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{3} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$

$$\sin\frac{5}{3}\pi = \sin\left(2\pi - \frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{11}{6}\pi = \cos\left(2\pi - \frac{\pi}{6}\right) = \cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

3 Verifica l'identità $\frac{\sin\alpha\cos\alpha}{2} + 2\tan\alpha\cos^2\alpha - (\sin\alpha + \cos\alpha)^2 = \frac{\tan\alpha}{2}(\overbrace{1 - \sin^2\alpha}^{\cos^2\alpha}) - 1.$

/15

$$\frac{\sin\alpha\cos\alpha}{2} + 2\tan\alpha\cos^2\alpha - (\underbrace{\sin^2\alpha + \cos^2\alpha}_{1} + 2\sin\alpha\cos\alpha) = \frac{\tan\alpha}{2}\cos^2\alpha - 1$$

$$\frac{\sin\alpha\cos\alpha}{2} \cancel{+2\frac{\sin\alpha}{\cos\alpha}\cdot\cos^2\alpha} - 1 \cancel{- 2\sin\alpha\cos\alpha} = \frac{\sin\alpha}{2\cos\alpha}\cdot\cos^2\alpha - 1$$

$$\frac{\sin\alpha\cos\alpha}{2} - 1 = \frac{\sin\alpha\cos\alpha}{2} - 1 \qquad \text{OK!!}$$