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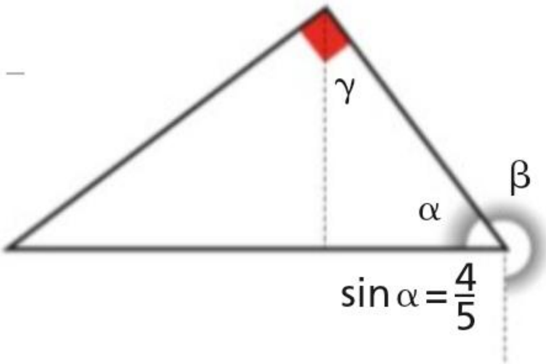


Trova:
$\sin\beta$, $\tan\gamma$, $\cos\left(\frac{\pi}{2}-\gamma\right)$.

$\left[-\frac{3}{5}; \frac{3}{4}; \frac{3}{5}\right]$

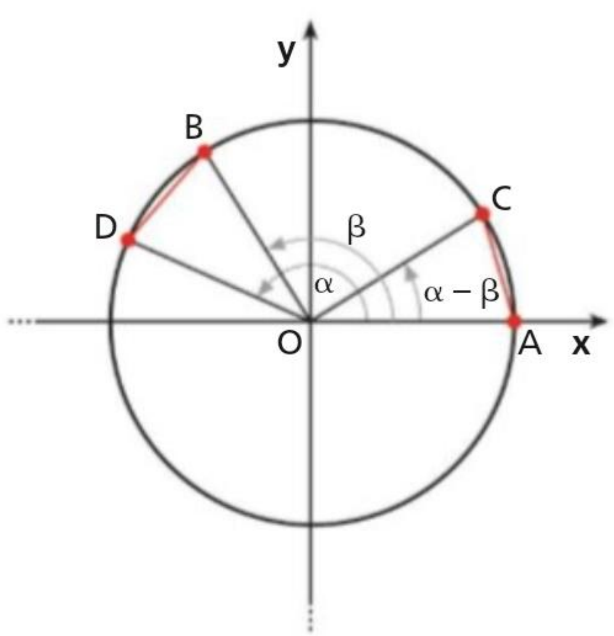
$$\sin\beta = \sin\left(\frac{3}{2}\pi - \alpha\right) =$$
$$= \sin\left(\pi + \frac{\pi}{2} - \alpha\right) =$$
$$= -\sin\left(\frac{\pi}{2} - \alpha\right) = -\cos\alpha =$$
$$= -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\left(\frac{4}{5}\right)^2} =$$
$$= -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\tan\gamma = \frac{\sin\gamma}{\cos\gamma} = \frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)} = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\cos\left(\frac{\pi}{2}-\gamma\right) = \cos\alpha = \frac{3}{5}$$

FORMULE DI ADDIZIONE E SOTTRAZIONE

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$



$A(1,0) \qquad C(\cos(\alpha-\beta), \sin(\alpha-\beta))$

$B(\cos\beta, \sin\beta) \qquad D(\cos\alpha, \sin\alpha)$

$\overline{AC} = \overline{BD} \Rightarrow \overline{AC}^2 = \overline{BD}^2$

$$[1-\cos(\alpha-\beta)]^2 + [0-\sin(\alpha-\beta)]^2 =$$
$$= [\cos\beta - \cos\alpha]^2 + [\sin\beta - \sin\alpha]^2$$

$$1 - 2\cos(\alpha-\beta) + \underbrace{\cos^2(\alpha-\beta) + \sin^2(\alpha-\beta)}_{1} = \cos^2\beta + \cos^2\alpha - 2\cos\alpha\cos\beta + \sin^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta$$

1 1

$$\cancel{2} - 2\cos(\alpha-\beta) = \cancel{2} - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta \rightarrow \text{DIVIDO PER } -2$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos 15^\circ = \cos(45^\circ-30^\circ)=\cos 45^\circ\cdot\cos 30^\circ+\sin 45^\circ\sin 30^\circ=$$

$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}\qquad\left(\cos\frac{\pi}{12}\right)$$

$$\cos(\alpha+\beta)=\cos(\alpha-(-\beta))=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)=$$

$$=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)=$$

$$=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta+\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta=$$

$$=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin(\alpha+(-\beta))=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)=$$

$$=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\sin 75^\circ=\sin(30^\circ+45^\circ)=\sin 30^\circ\cos 45^\circ+\sin 45^\circ\cos 30^\circ=$$

$$=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=$$

$\alpha+\beta\neq\frac{\pi}{2}+k\pi$

$\alpha,\beta\neq\frac{\pi}{2}+k\pi$

$$=\frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}+\dfrac{\sin\beta\cancel{\cos\alpha}}{\cos\alpha\cancel{\cos\alpha}}}{\dfrac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\tan(\alpha+(-\beta))=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

28 $\sin\left(\frac{2}{3}\pi-\alpha\right)+\sin\left(\alpha+\frac{5}{6}\pi\right)-\cos\left(\frac{\pi}{3}+\alpha\right)=$

$\left[\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha\right]$

$$=\sin\frac{2}{3}\pi\cos\alpha-\cos\frac{2}{3}\pi\sin\alpha+\sin\alpha\cos\frac{5}{6}\pi+\sin\frac{5}{6}\pi\cos\alpha$$

$$-\left[\cos\frac{\pi}{3}\cos\alpha-\sin\frac{\pi}{3}\sin\alpha\right]=$$

$$=\frac{\sqrt{3}}{2}\cos\alpha-\left(-\frac{1}{2}\right)\sin\alpha+\sin\alpha\left(-\frac{\sqrt{3}}{2}\right)+\cancel{\frac{1}{2}\cos\alpha}-\cancel{\frac{1}{2}\cos\alpha}+\frac{\sqrt{3}}{2}\sin\alpha=$$

$$=\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha-\cancel{\frac{\sqrt{3}}{2}\sin\alpha}+\cancel{\frac{\sqrt{3}}{2}\sin\alpha}=\boxed{\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha}$$

60 $\sin\left(\frac{\pi}{3}+\arccos\frac{4}{5}\right)=$ $\left[\frac{4\sqrt{3}+3}{10}\right]$

$$=\sin\frac{\pi}{3}\cos\left(\arccos\frac{4}{5}\right)+\sin\left(\arccos\frac{4}{5}\right)\cos\frac{\pi}{3}=$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{4}{5}+\sqrt{1-\cos^2\left(\arccos\frac{4}{5}\right)}\cdot\frac{1}{2}=\frac{2\sqrt{3}}{5}+\sqrt{1-\frac{16}{25}}\cdot\frac{1}{2}=$$

$$=\frac{2\sqrt{3}}{5}+\sqrt{\frac{9}{25}}\cdot\frac{1}{2}=\frac{2\sqrt{3}}{5}+\frac{3}{5}\cdot\frac{1}{2}=\boxed{\frac{4\sqrt{3}+3}{10}}$$

64 $\tan\left[\arctan\frac{12}{5} - \arcsin\left(-\frac{5}{13}\right)\right]$ [non esiste]

Devo controllare che $\arctan\frac{12}{5} - \arcsin\left(-\frac{5}{13}\right) \neq \frac{\pi}{2} + k\pi$

Provo comunque ad applicare la formula $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$$\tan\left(\arctan\frac{12}{5}\right) = \frac{12}{5}$$

$$\tan\left(\arcsin\left(-\frac{5}{13}\right)\right) = \frac{\sin\left(\arcsin\left(-\frac{5}{13}\right)\right)}{\cos\left(\arcsin\left(-\frac{5}{13}\right)\right)} = \frac{-\frac{5}{13}}{\sqrt{1-\sin^2\left(\arcsin\left(-\frac{5}{13}\right)\right)}} =$$

$$= \frac{-\frac{5}{13}}{\sqrt{1-\frac{25}{169}}} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$$

La formula NON è applicabile perché al denominatore ci sarebbe 0:

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\underbrace{\frac{12}{5}}_{\tan\alpha}\underbrace{\left(-\frac{5}{12}\right)}_{\tan\beta}}$$

Significa che $\alpha-\beta = \frac{\pi}{2}+k\pi$, dunque $\tan(\alpha-\beta)$ NON ESISTE

OSSERVAZIONE

Se $\alpha,\beta \neq k\frac{\pi}{2}$ e $\tan\alpha\cdot\tan\beta = -1$, allora $\alpha-\beta = \frac{\pi}{2}+k\pi$. Infatti:

$$\tan\alpha\cdot\tan\beta = -1 \Rightarrow \tan\alpha = -\frac{1}{\tan\beta} \Rightarrow \tan\alpha = -\cot\beta$$

$$\Rightarrow \tan\alpha = \tan\left(\frac{\pi}{2}+\beta\right) \Rightarrow \alpha = \frac{\pi}{2}+\beta+k\pi \Rightarrow \alpha-\beta = \frac{\pi}{2}+k\pi$$