31 $\sin^2(\alpha - 150°) + \cos^2(\alpha + 330°) - 1 =$

$$= \left[\sin\alpha \cos 150° - \sin 150° \cos\alpha\right]^2 + \left[\cos\alpha \cos 330° - \sin\alpha \sin 330°\right]^2 - 1 =$$

$$= \left[\sin\alpha\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\cos\alpha\right]^2 + \left[\cos\alpha \cdot \frac{\sqrt{3}}{2} - \sin\alpha\left(-\frac{1}{2}\right)\right]^2 - 1 =$$

$$= \frac{3}{4}\sin^2\alpha + \frac{1}{4}\cos^2\alpha + \frac{\sqrt{3}}{2}\sin\alpha\cos\alpha + \frac{3}{4}\cos^2\alpha + \frac{1}{4}\sin^2\alpha + \frac{\sqrt{3}}{2}\sin\alpha\cos\alpha - 1 =$$

$$\frac{3}{4}\left(\sin^2\alpha + \cos^2\alpha\right) = \frac{3}{4} \qquad \frac{1}{4}$$

$$= \frac{3}{4} + \frac{1}{4} - 1 + \sqrt{3}\sin\alpha\cos\alpha = \boxed{\sqrt{3}\sin\alpha\cos\alpha}$$

33 $\sin\left(\frac{7}{6}\pi - \alpha\right) \cdot \cos\left(\alpha - \frac{\pi}{3}\right) - \frac{1}{2}\cos^2\alpha =$

$$= \left[\sin\frac{7}{6}\pi \cos\alpha - \cos\frac{7}{6}\pi \sin\alpha\right] \cdot \left[\cos\alpha \cos\frac{\pi}{3} + \sin\alpha \sin\frac{\pi}{3}\right] - \frac{1}{2}\cos^2\alpha =$$

$$= \left[-\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right]\left[\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right] - \frac{1}{2}\cos^2\alpha =$$

$$= \frac{3}{4}\sin^2\alpha - \frac{1}{4}\cos^2\alpha - \frac{1}{2}\cos^2\alpha = \frac{3}{4}\left(1 - \cos^2\alpha\right) - \frac{1}{4}\cos^2\alpha - \frac{1}{2}\cos^2\alpha =$$

$$= \frac{3}{4} - \frac{3}{4}\cos^2\alpha - \frac{1}{4}\cos^2\alpha - \frac{1}{2}\cos^2\alpha = \frac{3}{4} - \frac{3}{2}\cos^2\alpha =$$

$$= \frac{3}{4}\left(1 - 2\cos^2\alpha\right) = -\frac{3}{4}\left(2\cos^2\alpha - 1\right) = -\frac{3}{4}\cos 2\alpha$$

FORMULA DI DUPLICAZIONE

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\cos 2\alpha = \cos(\alpha + \alpha) = \cos\alpha \cdot \cos\alpha - \sin\alpha \cdot \sin\alpha =$$

$$= \cos^2\alpha - \sin^2\alpha = \cos^2\alpha - (1 - \cos^2\alpha) =$$

$$= 2\cos^2\alpha - 1 = 2(1 - \sin^2\alpha) - 1 = 2 - 2\sin^2\alpha - 1$$

$$= 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= 2\cos^2\alpha - 1$$
$$= 1 - 2\sin^2\alpha$$

FORMULE DI DUPLICAZIONE

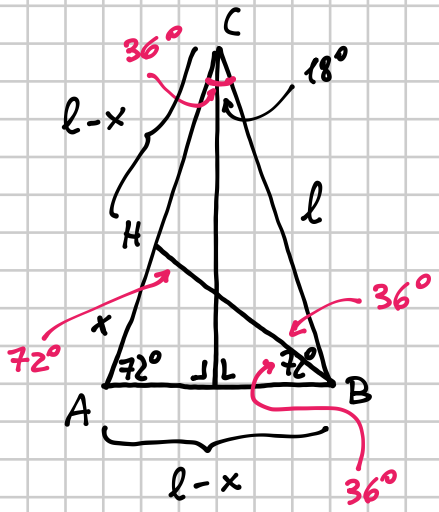
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\tan 2\alpha = \tan(\alpha + \alpha) = \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha \cdot \tan\alpha} = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$2\alpha \neq \frac{\pi}{2} + k\pi \Rightarrow \alpha \neq \frac{\pi}{4} + k\frac{\pi}{2}$$

$$\alpha \neq \frac{\pi}{2} + k\pi$$

SENO DELL'ANGOLO DI 18°



Per similitudine fra i triangoli ABC e ABH si ha:

$$(l-x):x = l:(l-x)$$

$$\overline{AB} : \overline{AH} = \overline{BC} : \overline{AB}$$

$$\Downarrow$$

$$(l-x)^2 = lx$$

$$l^2 + x^2 - 2lx = lx \Rightarrow x^2 - 3lx + l^2 = 0$$

$$\Delta = 9l^2 - 4l^2 = 5l^2$$

$$x = \frac{3l \pm \sqrt{5}l}{2} = \frac{3 \pm \sqrt{5}}{2} l$$

$$x = \frac{3+\sqrt{5}}{2} l$$

$$l - x = l - \frac{3+\sqrt{5}}{2} l = \frac{2-3-\sqrt{5}}{2} l < 0 \Rightarrow$$ la SOLUZIONE CON + NON È ACCETTABILE

$$x = \frac{3-\sqrt{5}}{2} l$$

$$l - x = \frac{2-3+\sqrt{5}}{2} l = \frac{\sqrt{5}-1}{2} l > 0$$

Dal triangolo si ha che $l \cdot \sin 18° = \frac{l-x}{2} \Rightarrow \sin 18° = \frac{l-x}{2l}$

$$\Rightarrow \sin 18° = \frac{\frac{\sqrt{5}-1}{2}\cancel{l}}{2\cancel{l}} = \frac{\sqrt{5}-1}{4}$$

62 $\cos\left[\arccos\frac{12}{13}-\arcsin\left(-\frac{4}{5}\right)\right]=$ $\left[\frac{16}{65}\right]$

$=\cos\arccos\frac{12}{13}\cdot\cos\left(\arcsin\left(-\frac{4}{5}\right)\right)+\sin\left(\arccos\frac{12}{13}\right)\sin\left(\arcsin\left(-\frac{4}{5}\right)\right)=$

$=\frac{12}{13}\underbrace{\sqrt{1-\left(-\frac{4}{5}\right)^2}}_{\sqrt{1-\sin^2\left(\arcsin\left(-\frac{4}{5}\right)\right)}}+\sqrt{1-\left(\frac{12}{13}\right)^2}\cdot\left(-\frac{4}{5}\right)=$

$=\frac{12}{13}\sqrt{1-\frac{16}{25}}+\sqrt{1-\frac{144}{169}}\cdot\left(-\frac{4}{5}\right)=\frac{12}{13}\sqrt{\frac{9}{25}}+\sqrt{\frac{25}{169}}\left(-\frac{4}{5}\right)=$

$=\frac{12}{13}\cdot\frac{3}{5}+\frac{5}{13}\cdot\left(-\frac{4}{5}\right)=\frac{36}{65}-\frac{20}{65}=\boxed{\frac{16}{65}}$