Formule di bisezione

$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}\qquad \sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad \tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\quad(\alpha\neq\pi+2k\pi)$$

$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}\quad \text{con } \alpha\neq\pi+2k\pi$$

$$\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}\quad \text{con } \alpha\neq k\pi$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 \qquad \cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 \qquad \cos\alpha = 1-2\sin^2\frac{\alpha}{2}$$

$$2\cos^2\frac{\alpha}{2} = 1+\cos\alpha \qquad 2\sin^2\frac{\alpha}{2} = 1-\cos\alpha$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \qquad \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad (\alpha\neq\pi+2k\pi)$$

$$\cos\alpha\neq-1$$

$$\alpha\neq\pi+2k\pi$$

$$\frac{\alpha}{2}\neq\frac{\pi}{2}+k\pi$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad \alpha \neq \pi + 2k\pi$$

$$\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}\cdot\frac{1+\cos\alpha}{1+\cos\alpha} = \frac{1-\cos^2\alpha}{(1+\cos\alpha)^2} = \frac{\sin^2\alpha}{(1+\cos\alpha)^2}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$$

$\sin\alpha$ ha lo stesso segno di $\tan\frac{\alpha}{2}$

SEMPRE >0

$\alpha \in [0, 2\pi]$

$$\sin\alpha > 0 \iff 0 < \alpha < \pi \iff 0 < \frac{\alpha}{2} < \frac{\pi}{2} \iff \tan\frac{\alpha}{2} > 0$$

$$\sin\alpha < 0 \iff \pi < \alpha < 2\pi \iff \frac{\pi}{2} < \frac{\alpha}{2} < \pi \iff \tan\frac{\alpha}{2} < 0$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} \qquad \alpha \neq \pi + 2k\pi$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}\cdot\frac{1-\cos\alpha}{1-\cos\alpha} = \frac{\sin\alpha(1-\cos\alpha)}{1-\cos^2\alpha} = \frac{\cancel{\sin\alpha}(1-\cos\alpha)}{\sin^{\cancel{2}}\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

$\cos\alpha \neq 1 \quad \alpha \neq 2k\pi$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} \qquad \alpha \neq k\pi$$

207 $\tan\frac{\alpha}{2} - \frac{\sin\alpha}{\cos^2\frac{\alpha}{2}} =$ $\left[-\frac{\sin\alpha}{1+\cos\alpha}\right]$

$$= \tan\frac{\alpha}{2} - \frac{\sin\alpha}{\frac{1+\cos\alpha}{2}} = \tan\frac{\alpha}{2} - \frac{2\sin\alpha}{1+\cos\alpha} =$$

$$= \frac{\sin\alpha}{1+\cos\alpha} - \frac{2\sin\alpha}{1+\cos\alpha} = \frac{\sin\alpha - 2\sin\alpha}{1+\cos\alpha} = -\frac{\sin\alpha}{1+\cos\alpha}$$

218 $2\sin^2\frac{\alpha}{2}\cdot\cot^2\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} =$ $\left[\frac{1+\cos\alpha}{2}\right]$

$$= 2\cancel{\sin^2\frac{\alpha}{2}}\cdot\frac{\cos^2\frac{\alpha}{2}}{\cancel{\sin^2\frac{\alpha}{2}}} - \cos^2\frac{\alpha}{2} = 2\cos^2\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} =$$

$$= \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

Verificare l'identità

221 $\cot^2\frac{\alpha}{2} = 4\cot\alpha\cdot\csc\alpha + \tan^2\frac{\alpha}{2}$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = 4\frac{\cos\alpha}{\sin\alpha}\cdot\frac{1}{\sin\alpha} + \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha(1+\cos\alpha)+\sin^2\alpha(1-\cos\alpha)}{\sin^2\alpha(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha\,(1+\cos\alpha) + \sin^2\alpha\,(1-\cos\alpha)}{\sin^2\alpha\,(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha\,(1+\cos\alpha) + (1-\cos^2\alpha)(1-\cos\alpha)}{\sin^2\alpha\,(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha\,(1+\cos\alpha) + (1+\cos\alpha)(1-\cos\alpha)(1-\cos\alpha)}{\sin^2\alpha\,(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{\cancel{(1+\cos\alpha)}\left[4\cos\alpha + (1-\cos\alpha)^2\right]}{\sin^2\alpha\,\cancel{(1+\cos\alpha)}}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{4\cos\alpha + 1 + \cos^2\alpha - 2\cos\alpha}{1-\cos^2\alpha}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{1 + \cos^2\alpha + 2\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{(1+\cos\alpha)^{\cancel{2}}}{(1-\cos\alpha)\cancel{(1+\cos\alpha)}}$$

$$\frac{1+\cos\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{1-\cos\alpha}$$ OK!