216 $\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2-\cos^2\frac{\alpha}{2}+\frac{1}{2}\cos\alpha$

$\left[\frac{1}{2}+\sin\alpha\right]$

$$\underbrace{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}_{1}+\underbrace{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}_{\sin\alpha}-\frac{1+\cos\alpha}{2}+\frac{1}{2}\cos\alpha=$$

$$=1+\sin\alpha-\frac{1}{2}-\cancel{\frac{1}{2}\cos\alpha}+\cancel{\frac{1}{2}\cos\alpha}=\boxed{\frac{1}{2}+\sin\alpha}$$

217 $\dfrac{1}{1-\tan\frac{\alpha}{2}}+\dfrac{1}{1+\tan\frac{\alpha}{2}}=$

$\left[\dfrac{1+\cos\alpha}{\cos\alpha}\right]$

$$=\frac{1+\cancel{\tan\frac{\alpha}{2}}+1-\cancel{\tan\frac{\alpha}{2}}}{\left(1-\tan\frac{\alpha}{2}\right)\left(1+\tan\frac{\alpha}{2}\right)}=\frac{2}{1-\tan^2\frac{\alpha}{2}}=$$

$$=\frac{2}{1-\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{2}{\frac{\cancel{1}+\cos\alpha-\cancel{1}+\cos\alpha}{1+\cos\alpha}}=\frac{\cancel{2}}{\frac{\cancel{2}\cos\alpha}{1+\cos\alpha}}=$$

$$=\boxed{\frac{1+\cos\alpha}{\cos\alpha}}$$

223 $$\frac{1+\sin\alpha}{\cos\alpha} = \frac{\cot\frac{\alpha}{2}+1}{\cot\frac{\alpha}{2}-1}$$

$$\frac{1+\sin\alpha}{\cos\alpha} = \frac{\frac{1}{\tan\frac{\alpha}{2}}+1}{\frac{1}{\tan\frac{\alpha}{2}}-1}$$

$$\dots = \frac{\frac{1+\tan\frac{\alpha}{2}}{\cancel{\tan\frac{\alpha}{2}}}}{\frac{1-\tan\frac{\alpha}{2}}{\cancel{\tan\frac{\alpha}{2}}}} = \frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}} = \frac{1+\frac{\sin\alpha}{1+\cos\alpha}}{1-\frac{\sin\alpha}{1+\cos\alpha}} =$$

$$= \frac{\frac{1+\cos\alpha+\sin\alpha}{\cancel{1+\cos\alpha}}}{\frac{1+\cos\alpha-\sin\alpha}{\cancel{1+\cos\alpha}}} = \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha-\sin\alpha}$$

$$\frac{1+\sin\alpha}{\cos\alpha} = \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha-\sin\alpha}$$

$$(1+\sin\alpha)(1+\cos\alpha-\sin\alpha) = \cos\alpha(1+\cos\alpha+\sin\alpha)$$

$$\underbrace{1+\cos\alpha-\cancel{\sin\alpha}+\cancel{\sin\alpha}+\sin\alpha\cos\alpha-\sin^2\alpha}_{\cos^2\alpha} = \cos\alpha+\cos^2\alpha+\cos\alpha\sin\alpha$$

$$\cos\alpha+\cos^2\alpha+\sin\alpha\cos\alpha = \cos\alpha+\cos^2\alpha+\sin\alpha\cos\alpha \quad \text{ok!}$$

FORMULE PARAMETRICHE

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} \qquad \cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$$

$$\sin\alpha = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}}$$

$$\sin\alpha = \frac{\dfrac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} \qquad \boxed{\begin{gathered}\frac{\alpha}{2} \neq \frac{\pi}{2} + k\pi \\ \Downarrow \\ \alpha \neq \pi + 2k\pi\end{gathered}} \qquad \cos\alpha = \frac{\dfrac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{1 - \tan^2\frac{\alpha}{2}}{1 + \tan^2\frac{\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = t$$

$$\boxed{\sin\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2} \qquad \begin{gathered}\alpha \neq \pi + 2k\pi \\ t = \tan\frac{\alpha}{2}\end{gathered}}$$

264 $\dfrac{2\sin\alpha + 3\cos\alpha}{1+\cos\alpha} =$ $\left[\dfrac{4t+3-3t^2}{2}\right]$

SCRIVERE NELLA VARIABILE $t = \tan\frac{\alpha}{2}$

$\sin\alpha = \dfrac{2t}{1+t^2}$

$\cos\alpha = \dfrac{1-t^2}{1+t^2}$

$\alpha \neq \pi + 2k\pi$

$$= \frac{2\cdot\dfrac{2t}{1+t^2} + 3\cdot\dfrac{1-t^2}{1+t^2}}{1+\dfrac{1-t^2}{1+t^2}} =$$

$$= \frac{\dfrac{4t+3-3t^2}{\cancel{1+t^2}}}{\dfrac{1+\cancel{t^2}+1-\cancel{t^2}}{\cancel{1+t^2}}} = \frac{4t+3-3t^2}{2}$$

Determina i valori richiesti, utilizzando le informazioni.

312 $\cos(\alpha-\beta), \sin\frac{\beta}{2}$; $\sin\alpha = \frac{\sqrt{5}}{3}$, con $\underbrace{0<\alpha<\frac{\pi}{2}}_{\cos\alpha>0}$ e $\cos\beta = \frac{3}{5}$, con $\underbrace{\frac{3}{2}\pi<\beta<2\pi}_{\sin\beta<0}$. $\left[\frac{6-4\sqrt{5}}{15}; \frac{\sqrt{5}}{5}\right]$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{2}{3}\cdot\frac{3}{5} + \frac{\sqrt{5}}{3}\cdot\left(-\frac{4}{5}\right) = \frac{6}{15} - \frac{4\sqrt{5}}{15} =$$

$$= \frac{6-4\sqrt{5}}{15}$$

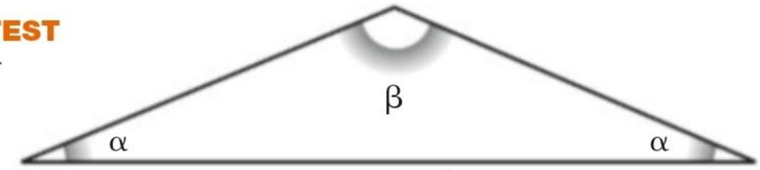
$$\boxed{\begin{array}{l} \cos\alpha = +\sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3} \\ \sin\beta = -\sqrt{1-\cos^2\beta} = -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5} \end{array}}$$

$$\sin\frac{\beta}{2} = +\sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\frac{3}{2}\pi < \beta < 2\pi$$

$$\Downarrow$$

$$\frac{3}{4}\pi < \frac{\beta}{2} < \pi \Rightarrow \sin\frac{\beta}{2} > 0$$



Nella figura, β è:

A $\pi - \arcsin \frac{120}{169}$.

B $\pi - \arccos \frac{12}{13}$.

C $\pi - 2 \arctan \frac{12}{5}$.

D $\pi - \arccos \frac{5}{13}$.

$$2\alpha + \beta = \pi$$
$$\Downarrow$$
$$\beta = \pi - 2\alpha$$

$$\alpha = \arcsin \frac{5}{13}$$

$$\beta = \pi - 2 \arcsin \frac{5}{13}$$

$$\sin\left(2 \arcsin \frac{5}{13}\right) = 2 \sin\left(\arcsin \frac{5}{13}\right) \cos\left(\arcsin \frac{5}{13}\right) =$$

$$= 2 \cdot \frac{5}{13} \cdot \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{10}{13} \cdot \sqrt{1 - \frac{25}{169}} = \frac{10}{13} \cdot \frac{12}{13} = \frac{120}{169}$$

$\downarrow$ applico l'arcsin

$$2 \arcsin \frac{5}{13} = \arcsin \frac{120}{169} \Rightarrow \beta = \pi - \arcsin \frac{120}{169}$$

OSSERVAZIONE

Per dedurre $2 \arcsin \frac{5}{13} = \arcsin \frac{120}{169}$ da $\sin\left(2 \arcsin \frac{5}{13}\right) = \frac{120}{169}$

si deve controllare che $0 < 2 \arcsin \frac{5}{13} < \frac{\pi}{2}$.

Infatti si ha:

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} > \frac{5}{13} \Rightarrow \frac{\pi}{4} > \arcsin \frac{5}{13}$$

FACILE DA VEDERE

$$\Downarrow$$

$$2 \arcsin \frac{5}{13} < \frac{\pi}{2}$$